Fourth International Conference
Modelling and Development of Intelligent Systems
October 28 - November 1, 2015
"Lucian Blaga" University
Sibiu - Romania

Optimization of Control Systems By PENDULAR Concept

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Abstract

The paper presents an inciting strategy in control, intire developed by the author, PENDULAR control. PENDULAR is the mnemonic of Pendulum Efficiency with Nonlinear Dynamics in Achievement of Robustness. The main idea is to optimize conventional structures using a nonlinear element on the feedback loop. That will transform a conventional control system into a variable structure system (VSS system). Having in view the simplicity of the control algorithm, a complete description of these systems, the study of stability and the Essential PENDULAR system (EPS) is presented in the paper. Simulation examples and experimental results show the efficiency of the PENDULAR concept.

1 Introduction

The word PENDULAR comes from the Romanian verb "a pendula" that defines the pendulum movement. PENDULAR (Pendulum Efficiency with Nonlinear Dynamics and Unconventional Law in Achievement of Robustness) systems are a class of nonlinear control systems introduced by the author in automatic control. Variable structure systems (VSS) are very interesting to be studied because often reveal surprises. Usually, the feedback control systems are closing the loop via a *negative feedback.* In this manner it is assumed the fact that the control system is robust (stable and effective even if different exogenous will disturb: reference variations, external disturbances, measurement noises, and plant uncertainties). The question is if the positive feedback is always bad for a control system?

A system exhibiting *positive feedback*, in response to *perturbation,* acts to increase the magnitude of the perturbation.In contrast, a system that responds to a perturbation in a way that reduces its effect is said to exhibit *negative feedback*. *Positive feedback often leads to exponential divergences or the exponential growth of oscillations*. Formally, a system in equilibrium in which there is positive feedback to any change from its current state is said to be in an unstable equilibrium. The magnitudes of the forces which act to move such a system away from its set point are an increasing function of the "distance" from the set point.*In the real world, positive feedback loops are always controlled eventual-*

ly by negative feedback or limiting effects of some sort. Figure 1 gives a simple view of this concept along with scientific terms and symbols.
Note that the response *y* is also called *system's behavior* or *performance*. The input *u* is often called the *control*.

Figure 1. Schematic diagram of a system with its input and output

As seen in Figure 1 one respects the causality principle.
The output $y(t)$ is related to the input $u(t)$ by the following equation:

$$y(t) = Tu(t) \tag{1}$$

where *T* is an operator applied to *u*.

In (1) three elements are involved: the input *u*, the system represented by the operator *T*, and the output *y*. In most of engineering problems *two of these three elements are given and we are asked to find the third one*. This observation is very important because the following three basic engineering problems arise: 1.*The analysis problem*. Here, we are given the input *u* and the system *T* and we are asked to determine the output *y*. 2.*The synthesis problem.* Here, we are given the input *u* and the output *y* and we are asked to determine the system *T*. *3.The measurement problem.* Here, we are given the system *T* and the output *y* and we are asked to measure the input *u*.

Definition 1.Given the system *T* and the output *y* known as the *desired response* we are asked to find an appropriate input signal *u*, such that, when this signal is applied to system *T*, the output of the system has to be the desired response *y*. The appropriate input signal *u* is called *control signal*.
From this definition it appears that the *control design problem* is in fact a signal synthesis problem: the synthesis of the *control signal u*. As it will be shown later, in practice, the control design problem is reduced to that of designing a controller. Control systems can be divided into two categories: *open-loops systems* and *closed-loops systems*.

Definition 2. An *open-loop system*, as shown in Figure 2, is a system whose input *u* does not depend on the output *y*, i.e., *u* is not a function of *y*.

Definition 3. A *closed-loop system*, Figure 3, is a system whose input *u* does depend on the output *y*, i.e., *u* is a function of *y*.In control systems, the control signal *u* is not the output of a signal generator, but the output of another new additional component that we add to the system under control. This new component is called *controller* and in special cases *regulator* or *compensator*. Furthermore, in control systems the controller is excited by an external signal *r* which is called the *reference* or *command* signal.

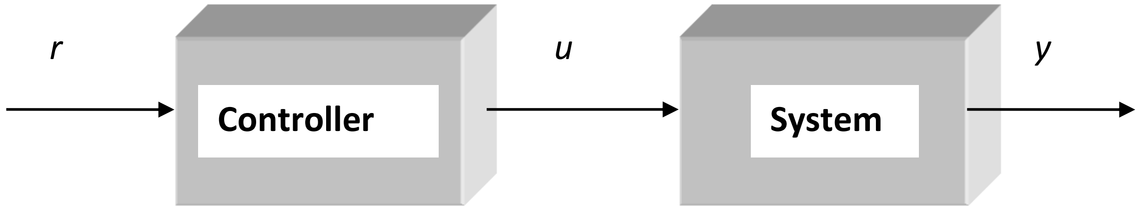


Figure 2 Open-loop system

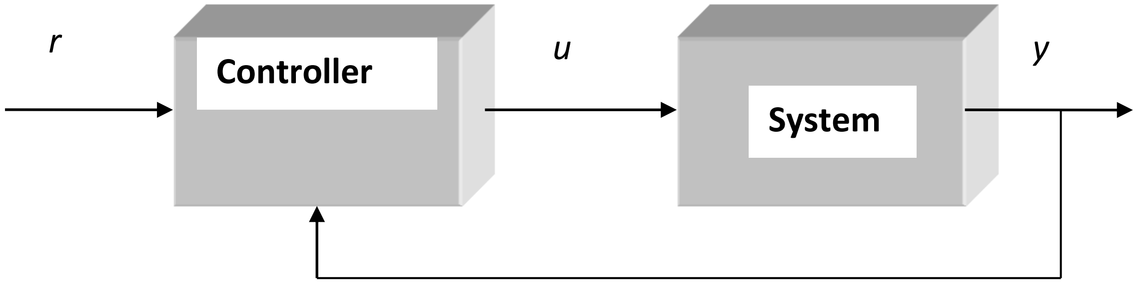
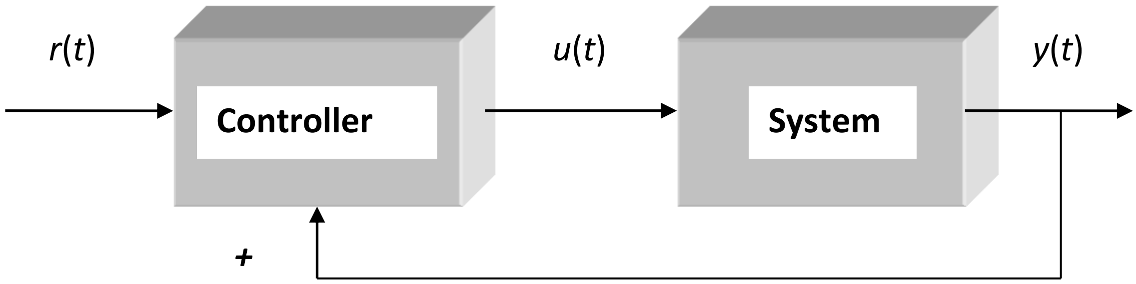


Figure 3 Closed-loop system

The reference signal r specified the desired performance. *That is, in control systems, we aim to design an appropriate controller such the desired output* y *follows the command signal* r *as close as possible*. In closed loop systems the controller is excited not only by reference signal r but also by the output y. Therefore, in this case the control signal u depends on both r and y.What is characteristic for a closed loop system? The answer to this question is very simple: the *feedback*. In fact, a closed loop control system is a *feedback system* because the following dependence can be written:

$$u(t) = u(r(t), y(u(t))) \tag{2}$$

The feedback is a permanent process of *comparison between what we want and what we get* and taking decisions. A positive feedback system is shown below.



For simplicity and for a clear understanding let consider that controller and systems are amplifiers with k_1 and k_2 .gains. Please remark the “+” sign!
Let suppose even more that $1=k_1=k_2$
Then:

$$y = k_2 u = k_2 k_1 (r + y) \Leftrightarrow y = \frac{k_2 k_1}{1 - k_2 k_1} r \to \infty \tag{3}$$

The relation (3) shows "that disaster" represented by infinity. That is mathematics. Intuitively, analyzing the dynamicity of feedback we have:
Step 1 .*r*=1, *f*=0, *u*=0, *y*=0. *Step 2* . *r*=1, *f*=0, *u*=1, *y*=0. *Step 3.* *r*=1, *f*=0, *u*=1, *y*=1. *Step 4.* *r*=1, *f*=1, *u*=1, *y*=1. *Step 5.r*=1, *f*=1, *u*=2, *y*=1. *Step 6.* *r*=1, *f*=1, *u*=2, *y*=2. *Step 7.* *r*=1, *f*=2, *u*=3, *y*=3 etc.

In the above considered steps *r* is *reference, f feedback*, *u command, y output*. One can see that *y increases and tends to infinity*! That means instability, chaos, disaster!

Besides, how *negative feedback* works? *Mutatis mutandis* the *negative feedback system* is depicted in Figure 4.

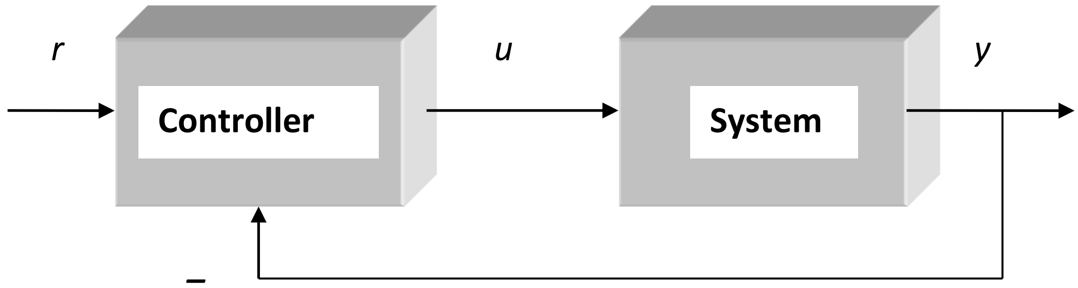


Figure 4. System with negative feedback

Supposing that the controller and systems are amplifiers with k_1 and k_2 .gains and 1=k_1=k_2

$$y = k_2 u = k_2 k_1 (r - y) \Leftrightarrow y = \frac{k_2 k_1}{1 + k_2 k_1} r = \frac{1}{2} \tag{4}$$

Similar: *Step 1 r*=1, *f*=0, *u*=0, *y*=0. *Step 2 r*=1, *f*=0, *u*=1/2, *y*=0. *Step 3 r*=1, *f*=1/2, *u*=1/2, *y*=1/2. *Step 4* *r*=1, *f*=1/2, *u*=1/2, *y*=1/2
So, *intuitively speaking, after 4 steps stability and stationary state can be obtained!*
Negative feedback is "good" while positive feedback with its cumulative effect is "bad"-this would be the natural conclusion expressed in the table below.

GOOD	BAD	COMPROMISE	PERFORMANCE
Negative feedback	*Positive feedback*	*Variable Structure System* (VSS)	*Integral index* IAE, ITAE, ISE, ITSE, etc
Stability	*Instability*	*Switching time ?*	*Minimization*

In fact, the PENDULAR concept philosophy is very simple. For example, let assume that one analyzes "*a level control system*" with "*automation at the level 0*". In other terms, a human operator supervises the water level in a tank. He turns off the tap whenever the water will reach the reference level. For efficiency, and if the tank volume is large, the operator does *not* proceed like this: turns on the tap on the drop by drop position and if the rising level approaches to the reference value turns off the tap. He turns on the tap at a large flow and when the level is near the reference level turns off the tap. Large

flow corresponds to positive feedback, small flow (drop by drop) corresponds to negative feedback. *Extending this very simple idea to the control loops the proposed system starts with positive feedback and at a certain time changes his structure becoming a negative (classic) control system*. In this manner the *PENDULAR control system* is defined.The proposed system will start with positive feedback and changes his structure becoming a negative (classic) control system at a *certain* time.
The paper consists in the following sections: introduction, pendular control system (PCS) (here pendular control principles are detailed), stability of PCS, Essential PCS (the search for simplicity), experimental results (made on a physical plant) and conclusions.

2 Pendular control system

Let the control system depicted inthe figure 5,

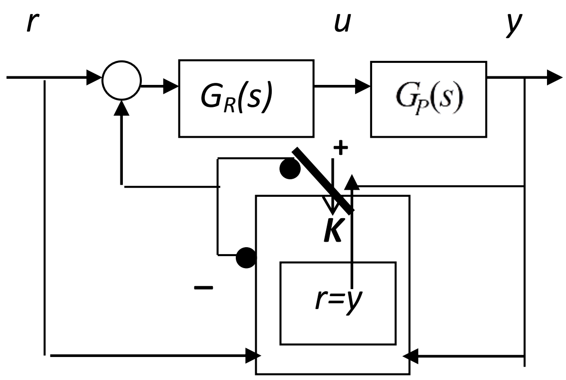


Figure 5 PENDULAR control system

obtained introducing a nonlinear element ***N*** on the conventional system feedback loop. The signals *r*, *e*, *u*, *y*, G_R and G_p are respectively the reference, error, command, output, the controller transfer function and the plant transfer function. ***N*** contains a decision block, a switch ***K***. Initially the switch is on "+" position. The decision block commands the switch ***K***, "**+**" to **"-"** for the very first time t_c. The system changes its structure at time t_c, $y(tc)=r$. The simplified control loop is depicted in Fig. 3:

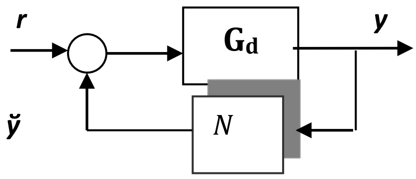


Figure 3 *Simplified control system*

where *Gd* is the open loop transfer function. The nonlinear element *N* is characterized by:

$$\tilde{y}(y)=\begin{cases}+y, & y\in[0,r]\\ -y, & y\in(r,\infty)\end{cases}, \tag{5}$$

Definition 4. *The nonlinear element N with characteristic (5) is called PENDULAR nonlinear element.*
N leads the system to the following behaviour:

• till the moment t_c, $\tilde{y}(y) = y, \quad y \in [0, r],\ y(t_c) = r$, system has a positive feedback with closed loop tranfer function: $G_{0+}(s) = \dfrac{G_R(s)G_p(s)}{1 - G_R(s)G_p(s)} = \dfrac{G_d(s)}{1 - G_d(s)}$, (6)

•for $t > t_c$, system has a negative feedback ($\tilde{y}(y) = -y$) *only if* $y(t) \geq r$. The closed loop transfer function is: $G_{0-}(s) = \dfrac{G_R(s)G_p(s)}{1 + G_R(s)G_p(s)} = \dfrac{G_d(s)}{1 + G_d(s)}$. (7)

However, $y(t), t \geq t_c$ represents the differential equation Cauchy problem for the negative feedback system (the conventional one), initial condition $y(t_c) = r$. If for $t_{c1} > t_c$, $y(t_{c_1}) = r$ and for $t > t_{c1}$, $y(t) < r$ the system will behave as positive feedback system (*K* comutes “-“ to “+”) and so on.

In this manner, the controlled variable *y*, may be considered as output signal for the positive feedback system, then at the moment t_c, after the very first commutation, output signal for the negative feedback system; eventually for the moment t_{c1}, again output signal for the positive feedback system,etc. till the controlled signal variable is stabilized at the value *r*.

The pendulum image for the output *y* comes from the movement between the two classes of differential equations solutions: S_+ (for positive feedback sub-system) and S_(for positive feedback sub-system) is represented in Fig 5.

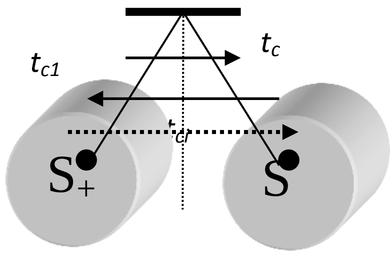


Figure5 *The y oscilatory “movement” scenario*

Definition 5. ***PENDULAR*** *control system (PCS) is the system obtained introducing a PENDULAR nonlinear element N on the feedback loop of a classical control system.*

Illustrative example. Let the disturbance *p* be an exogene signal applied as supplementary input to plant. Let assume that *r=1*, a proportional controller *k>0*, an integrator as plant and no disturbance (*p=0*). One obtains: $G_{0+}(s) = \dfrac{\frac{k}{s}}{1 - \frac{k}{s}} = \dfrac{k}{s-k}$; $G_{0-}(s) = \dfrac{\frac{k}{s}}{1 + \frac{k}{s}} = \dfrac{k}{s+k}$.

The system behaviour:

• $t \in [0, t_c]$.System starts with positive feedback $\dfrac{dy_+}{dt} - ky_+ = k,\ y_+(0) = 0,\ t \in [0, t_c]$

with the solution $y_+(t) = e^{kt} - 1,\ t \in [0, t_c]$ (8)

Time *tc* is given by $y_+(t_c) = 1 \Leftrightarrow e^{kt_c} - 1 = 1 \Leftrightarrow t_c = \dfrac{1}{k}\ln 2$.

From (8), $t \geq t_c \Rightarrow y_+(t) \geq 1 = r$, that means switching “+”to”-“

• $t > t_c$. System is with negative feedback: $\frac{dy_-}{dt} + ky_- = k,\ y_-(t_c+) = 1,\ t > t_c$,the solution:

$$\begin{cases} y_-(t) = 1 + ce^{-kt}, t > t_c \\ y_-(t_c+) = 1 \Rightarrow c = 0 \end{cases} \Leftrightarrow y_-(t) = 1, t > t_c. \tag{9}$$

The relation (8) shows that the system was stabilized only after a commutation and stays in this state if no disturbance is reported. The system global response is: $y(t) = \begin{cases} e^{kt} - 1, t \in [0, t_c] \\ 1, \quad t \in (t_c, \infty) \end{cases}$ (10)

Let suppose that for $t_p > t_c$ a *p*. step disturbance is applied.

$$\frac{dy_-}{dt} + ky_- = k + p,\ y_-(t_p) = 1,\ t \geq t_p.\ y_-(t) = \frac{k+p}{k} - \frac{p}{k} e^{k(t_p - t)}, t \geq t_p. \tag{11}$$

$$y_-^{/}(t) = pe^{k(t_p - t)}, t > t_p, \tag{12}$$

The derivative sign is given by *p* sign. For $p > 0 \Rightarrow y_-^{/}(t) > 0 \Rightarrow y_-(t) > y_-(t_p) = 1, \forall t > t_p$ (13)

system stays on negative feedback, if: $p < 0 \Rightarrow y_-^{/}(t) < 0 \Rightarrow y_-(t) < y_-(t_p) = 1, \forall t > t_p$, (14)

the system changes its structure on positive feedback.

The differential equation is: $\frac{dy_+}{dt} - ky_+ = k + p,\ y_+(t_p+) = 1,\ p < 0,\ t > t_p$ (15)

$$y_+(t) = \frac{2k+p}{k} e^{k(t-t_p)} - \frac{k+p}{k},\ p < 0, t > t_p \cdot y_+^{/}(t) = (2k+p)\, e^{k(t-t_p)}, t > t_p, p < 0, \tag{16}$$

and the derivative sign depends on (*2k+p*).
For *k>0*, one obtains the cases

$$-2k < p < 0 \Leftrightarrow 2k + p > 0 \Leftrightarrow y_+^{/}(t) > 0 \Rightarrow$$
$$y_+(t) > y_+(t_p+) = 1, \forall t > t_p$$

The system commutes for *p<0*, on positive feedback, then on negative feedback etc. In this case we have a infinite number of commutations the output is kept on the reference value *r=1*.

• $p = -2k \Rightarrow y_+(t) = 1, t > t_p$.

The system behaviour is as in the first case.

• $p < -2k \Leftrightarrow y_+^{/}(t) < 0 \Rightarrow y_+(t) < y_+(t_p+) = 1,\ t > t_p$.

The system stays on positive feedback the output decreases continuously, system is unstable. In conclusion: 1.*p>0,* system behaviour is identical with the conventional one. 2. $-2k \leq p < 0$, the PCS rejects the disturbance component instantaneously, 3.*p<2k*, the system becomes unstable.

Example. PCS with *k=1*, at $t_p=10$ sec disturbances *0.5*, *-0.5* are applied. With *Matlab-Simulink PCS response* (*y*) and conventional system (*y-)*are represented. The command *u* is also depicted One observe the chattering fenomenon (Fig.7,8,9)

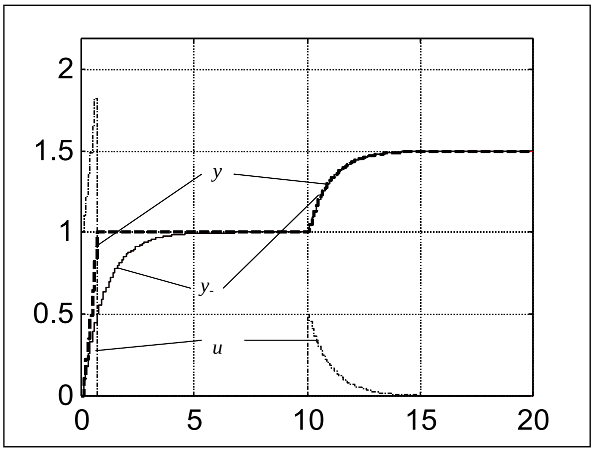


Fig 7**.** *Disturbance applied p=0.5*

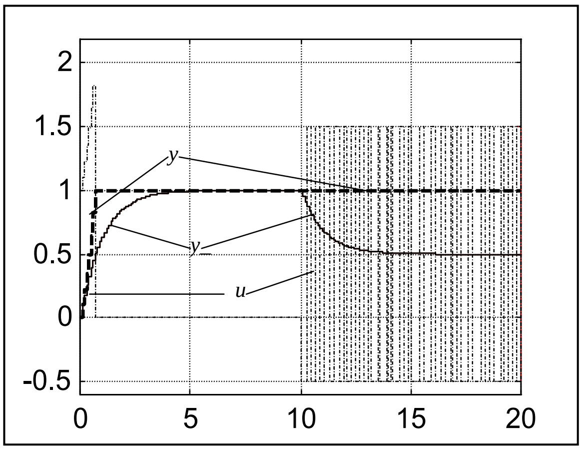


Fig. 8. *Disturbance applied: p=-0.5*

3 Stability of PCS

Definition 6. *The complex function F(s) is called real positive if:*
i.F(s) is analytic in $\{s : \mathrm{Re}\, s > 0\}$ *ii.* $\mathrm{Re}\, F(s) \geq 0$ for every s with $\mathrm{Re}\, s > 0$ *iii.* $F(\bar{s}) = \overline{F(s)}$ for every s with $\mathrm{Re}\, s > 0$.The function $F(s)$ analycity makes possible the replacement of *iii* with *iii'*. $F(s)$ is real for every s positive real.

Popov stability criterion
Theorem 1 (Popov) *The equilibrium state* $\bar{x} = 0$ *is globally asymptotic stable for the closed loop system (closed through h(t)) if*
1. $h(0) = 0$, *2.* $0 < \frac{h(y)}{y} < K_M, \forall y \neq 0,\ K_M > 0$ *3.there is* $\alpha \geq 0$, that $F(s) = (1 + \alpha s) G(s) + \frac{1}{K_M}$ *is a real positive function.*
Theorem 2 (Sandberg-Zames)
Let K_1 and K_2 two constants $K_2 > K_1$. *The equilibrium state* $\bar{x} = 0$ *is globally asymptotic stable for the closed loop system (closed through h(t)) if*
1. $h(0) = 0$, *2* $K_1 < \frac{h(y)}{y} < K_2, \forall y \neq 0,$ *3.* $F(s) = \frac{1 + K_2 G(s)}{1 + K_1 G(s)}$ *is a real positive function.*

Based on these two therems is very simple to analize the stability of PCS.

4 Essential PCS

Let PCS from *Fig.3*, with a *PI* controller and a innertial plant:

$$G_R(s) = k_R (1 + \frac{1}{T_i s}); \quad G_P(s) = \frac{k_P}{Ts + 1}; \quad T_i = T. \tag{17}$$

$$G_d(s) = G_R(s)\, G_P(s) = k_R\ (1 + \frac{1}{Ts}) \frac{k_P}{1 + Ts} = \frac{k_R k_P}{Ts} = \frac{k}{Ts}. \tag{18}$$

The closed loop transfer function: $G_{0-}(s)=\dfrac{G_d(s)}{1+G_d(s)}=\dfrac{k}{Ts+k}$ (19)

The system response for *r=1*:

$$h_-(t)=1-e^{-\frac{k}{T}t},\quad \forall t>0 \tag{20}$$

The response time 5% t_{r-} for the conventional system:

$$h_-(t_{r-})=0{,}95\Leftrightarrow e^{-\frac{k}{T}t_{r-}}=0.05\Leftrightarrow t_{r-}=\frac{T}{k}\ln 20\cong 3\frac{T}{k} \tag{21}$$

The PCS has for $t\in[0,t_c]$ transfer function

$$G_{0+}(s)=\frac{G_d(s)}{1-G_d(s)}\; G_{0+}(s)=\frac{\frac{k}{Ts}}{1-\frac{k}{Ts}}=\frac{k}{Ts-k},$$

unstable for $\forall k>0, T>0$.

The step response:

$$h_+(t)=L^{-1}\left\{\frac{G_{0+}(s)}{s}\right\}=L^{-1}\left\{\frac{k}{s(Ts-k)}\right\}=e^{\frac{k}{T}t}-1. \tag{22}$$

Time for the first commutation:

$$h_+(t_c)=e^{\frac{k}{T}t_c}-1=1\Leftrightarrow t_c=\frac{T}{k}\ln 2=0.693\frac{T}{k}. \tag{23}$$

"*Response time 5%*" :

$$h_+(t_{r_+})=0.95\Leftrightarrow t_{r_+}=\frac{T}{k}\ln 1.95\cong 0.667\frac{T}{k}\cdot \tag{24}$$

From (21) and (24) one obtains $t_{r_-}=\dfrac{T}{k}\ln 20\cong 3\dfrac{T}{k}\;\Rightarrow\; t_{r_-}\cong 4.48t_{r_+}$ (25)

Definition 7. *PCS that commutes one time when no disturbance applied is called essential pendular control system EPCS.*

Theorem 3 ***(PENDULAR essential theorem)****The system described above is EPCS. Its step response is given by:*

$$y(t)=\begin{cases}h_+(t), & t\in[0,t_c)\\ 1, & t\in[t_c,\infty)\end{cases}. \tag{26}$$

Demonstration. For $t\in[0,t_c)$, the response is given by (25).For $t\ge t_c$ the system response is given by the Cauchy problem: $T\dfrac{dh_-}{dt}+k\,h_-=k,\quad h_-(t_c)=1,\quad \forall t\ge t_c$. The general solution for (26) is:

$$h_-(t)=1+c\,e^{-\frac{k}{T}t},\quad t\ge t_c,\quad c\in R \tag{27}$$

with the initial condition from (26) one obtains *c=0*. Q.E.D.

In *Fig.10* the responses for the classic system (with conventional negative feedback) versus PCS system are depicted.

The responses were obtained by simulation via Matlab-Simulink environment. For simplicity *k=T=1*.

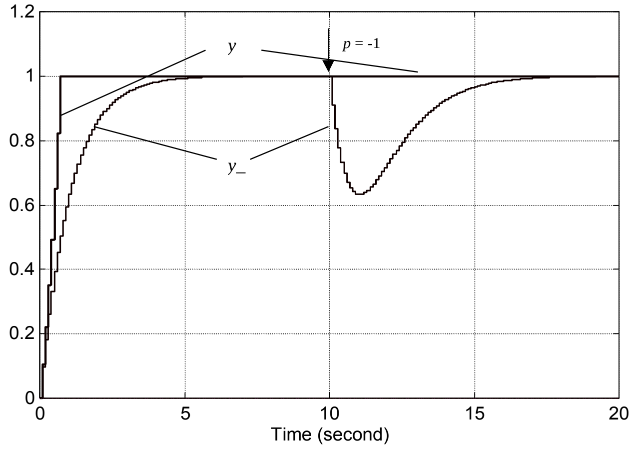


Fig.10 *Simulation results for k=T=1*

If a step disturbance will be applied (at *t=10* sec p=-1) an excelent behavior is reported for EPCS.From *Sandberg-Zames* theorem and the real positeveness theorem, the stability for PCS is very easy to prove.

Theorem 4.

PCS is globally asymptotic stable if $\frac{1-G(s)}{1+G(s)}$, *is a real positive function where G is the open loop trnsfer function.*

Very important observation*.Since G is Gr Gp and Gp is known one find the controller Gr.*

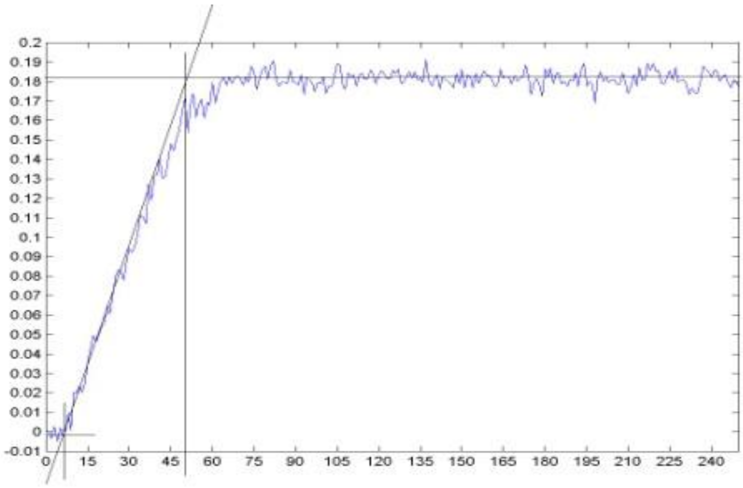
Application. EPCS case : Here $\frac{1-G(s)}{1+G(s)} = \frac{s-k}{s+k}$ and this function is obviously a real positive function.

5 Experimental setup

The set-up consists in a Feedback® Discovery Product for temperature and flow control. In this paper were made tests only for the flow control.The experiments have been done using the Real-Time Workshop from Matlab® Simulink®. The process has been identified as *an inertial order system with deadtime :*

$$G(s) = \frac{0.18}{2.8s+1}e^{-0.5s}$$

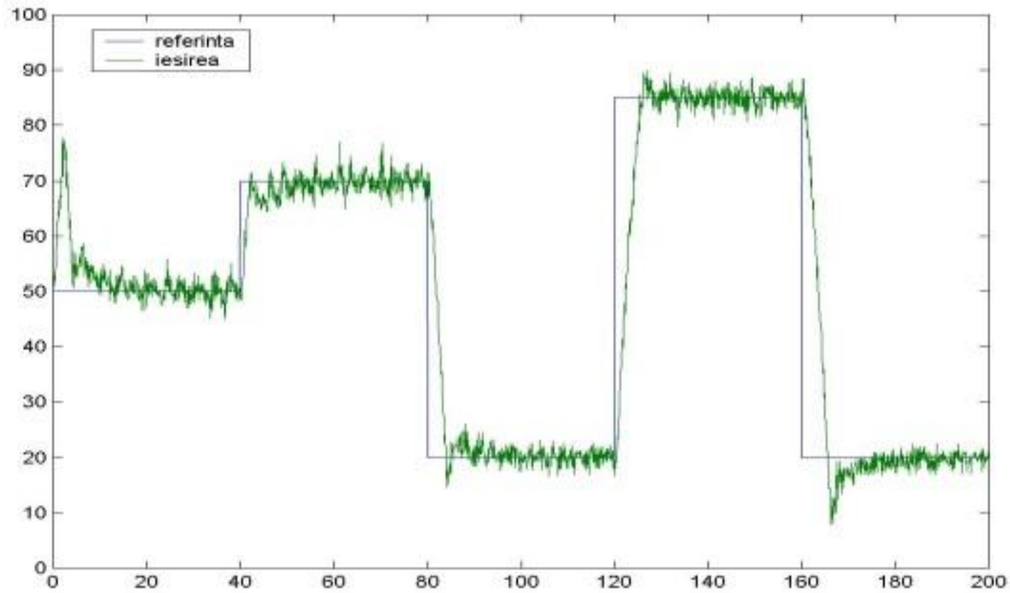
depicted below



Because the process is strongly affected by disturbances, the controller used is a PI type

$$G_R(s) = 2.76\left(1 + \frac{1}{4.66s}\right)$$

Creating a discrete-time model with the sample time of 0.1 seconds, using the PENDULAR control method, the below response is obtained.



6 Conclusions

The paper briefly presents the PENDULAR control systems. The PENDULAR control principles are sustained by a stability study. Essential PENDULAR Control systems were detailed and the simulation results are illustrative.The research is also sustained by experimental results made on different classes of systems. This new control methodology seems to have some impact over the control strategies.

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