Epistemic democracy with correlated and heterogeneous voters

(Preliminary report)

Marcus Pivato

THEMA, Université de Cergy-Pontoise, France

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Introduction

Epistemic social choice theory treats voting mechanisms as devices to determine the "ground truth" from information provided by voters:

- The *Condorcet Jury Theorem* (CJT) says that majority vote converges in probability to the correct answer as the population becomes large.
- The *Wisdom of Crowds* (WoC) *Principle* says that, if a large number of people estimate some numerical quantity, then the average of their estimates will converge, in probability, to the true value.

Problem: These results assume the voters are *stochastically independent*. This is obviously unrealistic. First, the voters are subject to common influences (e.g. newspapers). Second, they influence one another (e.g. through discussion).

Goals: (1) Extend the CJT and WoC to correlated, heterogeneous voters. (2) Obtain similar asymptotic results (with correlated, heterogeneous voters) for other epistemic social choice models.

(Previous work on CJT with correlations: by Boland, Prochan & Tong (1989), Berg (1993,1994,1996), Ladha (1992,1993,1995), Dietrich & List (2004) Kaniovski (2009,2010), Peleg & Zamir (2012), Dietrich & Spiekermann (2013).)

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Plan:

Part I. Mean partition rules.

Part II. Culture and correlation.

Part III. Social networks.

Part IV. DeGroot models of social influence.

Part I

Mean partition rules

Mean partition rules: definition

Let $\mathcal{I}$ be a set of individuals. Let $\mathcal{S}$ be a set of social alternatives.

A *mean partition rule* on $\mathcal{S}$ is a voting rule defined by a data structure $(\mathbb{V}, \mathcal{V}, \mathcal{C}, F)$, where:

- $\mathbb{V}$ is a real vector space (e.g. $\mathbb{V} = \mathbb{R}$ or $\mathbb{V} = \mathbb{R}^N$).
- $\mathcal{V} \subseteq \mathbb{V}$ is the set of possible votes which could be sent by each person.
- $\mathcal{C}$ is the convex hull of $\mathcal{V}$ in $\mathbb{V}$.
- $F : \mathcal{C} \longrightarrow \mathcal{S}$ is a surjective function such that for all $s \in \mathcal{S}$, the preimage set $F^{-1}\{s\}$ is a convex subset of $\mathcal{C}$.
- Given any profile $(\mathbf{v}_i)_{i \in \mathcal{I}}$ (where $\mathbf{v}_i \in \mathcal{V}$ for all $i \in \mathcal{I}$), the voting rule will choose the outcome $F\left(\frac{1}{I} \sum_{i \in \mathcal{I}} \mathbf{v}_i\right)$.

Note: If F is injective (so that $F^{-1}\{s\}$ is a singleton for all $s \in \mathcal{S}$), then the convexity condition is automatically satisfied.
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Example 1. *Simple majority rule*
Let $\mathcal{S} := \{\pm 1\}$ (two alternatives). Let $\mathbb{V}_{\rm maj} := \mathbb{R}$.
Let $\mathcal{V}_{\rm maj} := \{\pm 1\}$, so that $\mathcal{C} = [-1, 1]$.
Define $F_{\rm maj} : \mathcal{C} \longrightarrow \mathcal{S}$ by setting $F_{\rm maj}(c) := \operatorname{sign}(c)$ for all nonzero $c \in [-1, 1]$, while $F_{\rm maj}(0) := 1$ (an arbitrary tie-breaking rule).
This mean partition rule is equivalent to the *simple majority rule*.

Example 2. *Plurality rule*
Let $N \geq 2$. Let $\mathcal{S} := \{1, 2, \ldots N\}$ (a set of N alternatives). Let $\mathbb{V}_{\rm plu} := \mathbb{R}^N$.
For all $s \in [1 \ldots N]$, let $\mathbf{v}^s := (0, \ldots, 0, 1, 0, \ldots, 0)$ (with 1 in sth coordinate).
Let $\mathcal{V}_{\rm plu} := \{\mathbf{v}^1, \ldots, \mathbf{v}^N\}$ (a subset of $\mathbb{R}^N$).
Thus, $\mathcal{C} = \operatorname{conv}(\mathcal{V}_{\rm plu})$ is the unit simplex in $\mathbb{R}^N$.
Define $F_{\rm plu} : \mathcal{C} \longrightarrow \mathcal{S}$ by setting $F_{\rm plu}(\mathbf{c}) :=$ [the *maximal coordinate* of $\mathbf{c}$] for all $\mathbf{c} \in \mathcal{C}$ (with some arbitrary tie-breaking rule).
This mean partition rule is the standard *plurality rule*.

Example 3. *The average rule*
Let $N \geq 1$, and let $\mathcal{S}$ be a convex subset of $\mathbb{R}^N$.
Let $\mathcal{C} = \mathcal{V} = \mathcal{S}$, and let $F_{\rm ave} : \mathcal{C} \longrightarrow \mathcal{S}$ be the identity function.
In this mean partition rule, each voter declares an "ideal point" in $\mathcal{S}$, and the outcome is the *arithmetic average* of these ideal points.

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Example 4. *The generalized median rule.*
Let $(\mathcal{S}, d)$ be a metric space. Let $\mathbb{V}_{\mathrm{med}} := \mathbb{R}^{\mathcal{S}}$.

For all $s \in \mathcal{S}$, define $\mathbf{v}^s := (v_t^s)_{t\in\mathcal{S}} \in \mathbb{V}$, by setting $v_t^s := d(s,t)$, $\forall\ t \in \mathcal{S}$.

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Idea: Each voter chooses an "ideal point" s in $\mathcal{S}$ (represented by $\mathbf{v}^s$). The rule selects the point in $\mathcal{S}$ which *minimizes the average distance* to these ideal points.

Special cases: (a) If $\mathcal{S} \subset \mathbb{R}$, then this is the classic median rule.

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Example 4. *The generalized median rule.*
Let $(\mathcal{S}, d)$ be a metric space. Let $\mathbb{V}_{\rm med} := \mathbb{R}^{\mathcal{S}}$.

For all $s \in \mathcal{S}$, define $\mathbf{v}^s := (v^s_t)_{t\in\mathcal{S}} \in \mathbb{V}$, by setting $v^s_t := d(s,t)$, $\forall\ t \in \mathcal{S}$.

Let $\mathcal{V}_{\rm med} := \{\mathbf{v}^s\}_{s\in\mathcal{S}}$ (a subset of $\mathbb{V}$), and let $\mathcal{C}$ be the convex hull of $\mathcal{V}$.

Define $F_{\rm med} : \mathcal{C} \longrightarrow \mathcal{S}$ by setting $F_{\rm med}(\mathbf{c}) :=$ [the *minimal coordinate* of $\mathbf{c}$] for all $\mathbf{c} \in \mathcal{C}$ (with some arbitrary tie-breaking rule).

Idea: Each voter chooses an "ideal point" s in $\mathcal{S}$ (represented by $\mathbf{v}^s$). The rule selects the point in $\mathcal{S}$ which *minimizes the average distance* to these ideal points.

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Example 5. *Any scoring rule*
Let $\mathcal{S}$ be any set of alternatives.

Let $\mathbb{V}_{\rm scr} := \mathbb{R}^{\mathcal{S}}$, and let $\mathcal{V}$ be any subset of $\mathbb{V}_{\rm scr}$.

Any vote $\mathbf{v} = (v_s)_{s\in\mathcal{S}}$ in $\mathcal{V}$ assigns a "score" of v_s to each alternative in $\mathcal{S}$.

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The mean partition rule $(\mathbb{V}_{\rm scr}, \mathcal{V}, F_{\rm scr})$ is equivalent to a *scoring rule.*
Any scoring rule can be represented in this way.

All of the examples above are special cases of scoring rules.

Other scoring rules include the *Borda rule* and the *Approval voting* rule.

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Part II

Correlation and Culture

Correlated voters and collective behaviour model

Given a set $\mathcal{I}$ of individuals and a set $\mathcal{V}$ of votes, a *profile* is an element $\mathbf{V} = (\mathbf{v}_i)_{i \in \mathcal{I}}$ of $\mathcal{V}^{\mathcal{I}}$, which assigns a vote $\mathbf{v}_i$ to each individual i in $\mathcal{I}$.

Let $\mathcal{S}$ be the set of possible states of nature (the true state is unknown).

Definition. A *collective behaviour model* (CBM) is a function $\rho : \mathcal{S} \longrightarrow \Delta(\mathcal{V}^{\mathcal{I}})$.

Idea: Suppose the true state is $s \in \mathcal{S}$. Then for any profile $\mathbf{V} \in \mathcal{V}^{\mathcal{I}}$,

$$\rho(\mathbf{V}|s) \;=\; \begin{pmatrix}\text{the probability that we see the profile } \mathbf{V}, \\ \text{given that the true state of nature is } s\end{pmatrix}.$$

We cannot assume detailed knowledge of the CBM. We will only suppose that it arises from some family of CBMs with certain qualitative properties...

Definition. A *culture* on $\mathcal{V}$ is a sequence $\mathfrak{R} = (\mathcal{R}_I)_{I=1}^{\infty}$ where, for all $I \in \mathbb{N}$, $\mathcal{R}_I$ is a set of collective behaviour models ranging over $\mathcal{V}^I$.

Idea: A culture is not a description of a *single* society. It is a description of *all possible* societies, of all possible sizes.

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Correlation matrices

We must analyse the possible correlations between voters within a culture...

From now on, let $\langle \bullet, \bullet \rangle$ be an inner product structure on the vector space $\mathbb{V}$.

Let $I \in \mathbb{N}$. Let $\rho : \mathcal{S} \longrightarrow \Delta(\mathcal{V}^I)$ be a collective behaviour model.

Fix $s \in \mathcal{S}$, and let $\mathbf{V} = (\mathbf{v}_i)_{i \in \mathcal{I}}$ be a $\rho(s)$-random profile.

For any $i, j \in \mathcal{I}$, we define the *covariance* of voters i and j (given s) by

$$\mathrm{cov}(\mathbf{v}_i, \mathbf{v}_j) \quad := \quad \mathbb{E}[\langle \mathbf{v}_i - \widehat{\mathbf{v}}_i,\ \mathbf{v}_j - \widehat{\mathbf{v}}_j \rangle],$$

where $\widehat{\mathbf{v}}_i$ denotes the *expected value* of $\mathbf{v}_i$.

This measures the amount, *on average*, by which we can expect the errors of voters i and j to align in same direction in $\mathbb{V}$.

In particular, $\mathrm{var}[\mathbf{v}_i] := \mathrm{cov}(\mathbf{v}_i, \mathbf{v}_i) = \mathbb{E}\left[\|\mathbf{v}_i - \widehat{\mathbf{v}}_i\|^2\right]$ (the unreliability of i).

The *covariance matrix* of $\rho(s)$ is the $I \times I$ matrix $\mathrm{cov}[\rho(s)] := [b_{i,j}]_{i,j=1}^I$, where, for all $i, j \in [1 \ldots I]$, $\quad b_{i,j} := \mathrm{cov}(\mathbf{v}_i, \mathbf{v}_j)$.

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We must analyse the possible correlations between voters within a culture...

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Cultures and correlation structures

Definition. A *correlation structure* is a sequence $\mathfrak{B} = (\mathcal{B}_I)_{I=1}^{\infty}$, where, for all $I \in \mathbb{N}$, $\mathcal{B}_I$ is a collection of $I \times I$ symmetric, positive definite matrices.

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Note: For any $\rho \in \mathcal{R}_I$ and $s \in \mathcal{S}$, the matrix $\mathrm{cov}[\rho(s)]$ combines two sorts of information. Diagonal entries encode the "reliability" of individual voters. Off-diagonal entries are the correlations *between* voter errors.

$$\kappa(\mathbf{B}) := \frac{1}{I(I-1)} \sum_{i \neq j=1}^{I} b_{i,j} \;=\; [\text{average covariance } \textit{between} \text{ voter errors}].$$

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Notation. For any (covariance) matrix $\mathbf{B} \in \mathcal{B}_I$, define

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Continuity. $\mathbb{V}$ has an inner product $\langle \bullet, \bullet \rangle$. $\mathcal{S}$ has a metric d. There is a subset $\mathcal{C}' \subseteq \mathcal{C}$ such that F is uniformly continuous and surjective when restricted to $\mathcal{C}'$, with respect to $\langle \bullet, \bullet \rangle$ and d. For any $s \in \mathcal{S}$, the F-preimage of s inside $\mathcal{C}'$ is convex.

Identification. For any $I \in \mathbb{N}$, any $\rho \in \mathcal{R}_I$, and any $s \in \mathcal{S}$, if $(\mathbf{v}_i)_{i \in \mathcal{I}}$ is a $\rho(s)$-random profile, then for all $i \in [1 \ldots I]$, the *expected* value of $\mathbf{v}_i$ is in $F^{-1}\{s\} \cap \mathcal{C}'$.

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Let $\mathfrak{R} = (\mathcal{R}_I)_{I=1}^{\infty}$ be a culture, with correlation structure $(\mathcal{B}_I)_{I=1}^{\infty}$.

Definition. The culture $\mathfrak{R}$ is *sophogenic* ("wisdom-generating") for a mean partition rule $(\mathbb{V}, \mathcal{V}, \mathcal{C}, F)$ if it satisfies four properties....

CONTINUITY. $\mathbb{V}$ has an inner product $\langle \bullet, \bullet \rangle$. $\mathcal{S}$ has a metric d.
There is a subset $\mathcal{C}' \subseteq \mathcal{C}$ such that F is uniformly continuous and surjective when restricted to $\mathcal{C}'$, with respect to $\langle \bullet, \bullet \rangle$ and d.
For any $s \in \mathcal{S}$, the F-preimage of s inside $\mathcal{C}'$ is convex.

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Identification and Continuity

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Minimal average reliability

MINIMAL AVERAGE RELIABILITY. There is some constant $M > 0$ such that, for any $I \in \mathbb{N}$, and any $\mathbf{B} \in \mathcal{B}_I$, $\sigma(\mathbf{B}) \leq M$.

To understand this, note that is a weak form of the following condition:

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Idea: The variance of an individual's vote distribution is a measure of her (un)reliability: if the variance is large, then this voter has a high probability of picking the wrong answer.

MINIMAL RELIABILITY says that all voters meet at least some minimum standard of reliability.

MINIMAL AVERAGE RELIABILITY is a weak form of MINIMAL RELIABILITY. But now we allow some individuals to be *very* unreliable, as long as the *average* reliability is good.

Note: If $\mathcal{V}$ is bounded (e.g. finite), then MINIMAL RELIABILITY is always satisfied (since there is some M such that $\mathrm{var}(\rho) \leq M$ for any $\rho \in \Delta(\mathcal{V})$).

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This is the key condition. It says that voters' errors can be correlated in an arbitrary way, but as the society grows large, the *average* correlation between the errors of different voters must become small.

Example: A culture is *uncorrelated* if $b_{i,j} = 0$ for all $i \neq j$ and all $\mathbf{B} \in \mathcal{B}_I$.

In this case, ASYMPTOTICALLY WEAK AVERAGE CORRELATION is obviously satisfied. (More examples later...)

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Theorem 1. *Let* $(\mathbb{V}, \mathcal{V}, \mathcal{C}, F)$ *be a mean partition rule.*
Let $(\mathcal{R}_I)_{I=1}^{\infty}$ *be a sophogenic culture for* $(\mathbb{V}, \mathcal{V}, \mathcal{C}, F)$.
Let $s \in \mathcal{S}$ *(the true state). Let* $\mathcal{U} \subset \mathcal{S}$ *be an open set containing* s.
For all $I \in \mathbb{N}$, *let* $\rho_I \in \mathcal{R}_I$, *and let*

$$P_I \quad := \quad \mathrm{Prob}\left(F\left(\frac{1}{I}\sum_{i\in\mathcal{I}} \mathbf{v}_i\right) \in \mathcal{U} \;\middle|\; (\mathbf{v}_i)_{i=1}^{I} \text{ is a } \rho_I\text{-random profile} \right).$$

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Upshot: In a sophogenic culture, the outcome of F converges in probability to the true state of nature, as the voting population becomes large.

The Condorcet Jury Theorem is a special case of Theorem 1.

To see this, let $\mathcal{S} = \mathcal{V}_{\mathrm{maj}} := \{\pm 1\}$ and let F_{maj} be as in Example 1 above.

$$\underset{-1}{|} \;---\; \mathcal{C}_{-1} \;---\; \underset{-\epsilon/2}{|} \;-\; \underset{0}{|} \;-\; \underset{\epsilon/2}{|} \;---\; \mathcal{C}_{+1} \;---\; \underset{+1}{|}$$

Fix $\epsilon > 0$. Let $\mathcal{C}' := \mathcal{C}_{-1} \sqcup \mathcal{C}_{+1}$, where $\mathcal{C}_{-1} := [-1, -\frac{\epsilon}{2}]$ and $\mathcal{C}_{+1} := [\frac{\epsilon}{2}, 1]$. Then CONTINUITY is satisfied.

Let $\mathfrak{R} = (\mathcal{R}_I)_{I=1}^{\infty}$ be a culture such that, for all $I \in \mathbb{N}$, all $\rho \in \mathcal{R}_I$, and all $i \in [1 \ldots I]$, we have $\rho_i(s|s) > \frac{1}{2} + \epsilon$ for both $s \in \{\pm 1\}$.
[Here, $\rho_i : \mathcal{S} \longrightarrow \Delta(\mathcal{V})$ is projection of ρ onto ith coordinate.]

For any $\rho \in \mathcal{R}_I$, and for both $s \in \{\pm 1\}$ it is clear that $\mathbb{E}[\rho_i(s)] \in \mathcal{C}_s$ for all $i \in [1 \ldots I]$. Thus, $F_{\mathrm{maj}}(\mathbb{E}[\rho_i(s)]) = s$, so IDENTIFICATION is satisfied.

Finally, $\mathcal{V}_{\mathrm{maj}}$ is finite, so MINIMAL RELIABILITY is satisfied.

Thus, if $\mathfrak{R}$ satisfies ASYMPTOTICALLY WEAK AVERAGE CORRELATION, then Theorem 1 yields a Condorcet Jury Theorem for correlated, heterogeneous voters (similar to Ladha, 1992).

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The Condorcet Jury Theorem is a special case of Theorem 1.

To see this, let $\mathcal{S} = \mathcal{V}_{\rm maj} := \{\pm 1\}$ and let $F_{\rm maj}$ be as in Example 1 above.

$$\underset{-1}{|}\ ---\mathcal{C}_{-1}---\ \underset{-\epsilon/2}{|}\ -\ \underset{0}{|}\ -\ \underset{\epsilon/2}{|}\ ---\mathcal{C}_{+1}---\ \underset{+1}{|}$$

Fix $\epsilon > 0$. Let $\mathcal{C}' := \mathcal{C}_{-1} \sqcup \mathcal{C}_{+1}$, where $\mathcal{C}_{-1} := [-1, -\frac{\epsilon}{2}]$ and $\mathcal{C}_{+1} := [\frac{\epsilon}{2}, 1]$. Then CONTINUITY is satisfied.

Let $\mathfrak{R} = (\mathcal{R}_I)_{I=1}^{\infty}$ be a culture such that, for all $I \in \mathbb{N}$, all $\rho \in \mathcal{R}_I$, and all $i \in [1 \ldots I]$, we have $\rho_i(s|s) > \frac{1}{2} + \epsilon$ for both $s \in \{\pm 1\}$.
[Here, $\rho_i : \mathcal{S} \longrightarrow \Delta(\mathcal{V})$ is projection of ρ onto ith coordinate.]

For any $\rho \in \mathcal{R}_I$, and for both $s \in \{\pm 1\}$ it is clear that $\mathbb{E}[\rho_i(s)] \in \mathcal{C}_s$ for all $i \in [1 \ldots I]$. Thus, $F_{\rm maj}(\mathbb{E}[\rho_i(s)]) = s$, so IDENTIFICATION is satisfied.

Finally, $\mathcal{V}_{\rm maj}$ is finite, so MINIMAL RELIABILITY is satisfied.

Thus, if $\mathfrak{R}$ satisfies ASYMPTOTICALLY WEAK AVERAGE CORRELATION, then Theorem 1 yields a Condorcet Jury Theorem for correlated, heterogeneous voters (similar to Ladha, 1992).

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For any $\rho \in \mathcal{R}_I$, and for both $s \in \{\pm 1\}$ it is clear that $\mathbb{E}[\rho_i(s)] \in \mathcal{C}_s$ for all $i \in [1 \ldots I]$. Thus, $F_{\rm maj}\left(\mathbb{E}[\rho_i(s)]\right) = s$, so IDENTIFICATION is satisfied.

Finally, $\mathcal{V}_{\rm maj}$ is finite, so MINIMAL RELIABILITY is satisfied.

Thus, if $\mathfrak{R}$ satisfies ASYMPTOTICALLY WEAK AVERAGE CORRELATION, then Theorem 1 yields a Condorcet Jury Theorem for correlated, heterogeneous voters (similar to Ladha, 1992).

The Condorcet Jury Theorem is a special case of Theorem 1.

To see this, let $\mathcal{S} = \mathcal{V}_{\mathrm{maj}} := \{\pm 1\}$ and let F_{maj} be as in Example 1 above.

$$\underset{-1}{|} \quad ---\, \mathcal{C}_{-1} \,--- \quad \underset{-\epsilon/2}{|} \quad - \quad \underset{0}{|} \quad - \quad \underset{\epsilon/2}{|} \quad ---\, \mathcal{C}_{+1} \,--- \quad \underset{+1}{|}$$

Fix $\epsilon > 0$. Let $\mathcal{C}' := \mathcal{C}_{-1} \sqcup \mathcal{C}_{+1}$, where $\mathcal{C}_{-1} := [-1, -\frac{\epsilon}{2}]$ and $\mathcal{C}_{+1} := [\frac{\epsilon}{2}, 1]$. Then Continuity is satisfied.

Let $\mathfrak{R} = (\mathcal{R}_I)_{I=1}^{\infty}$ be a culture such that, for all $I \in \mathbb{N}$, all $\rho \in \mathcal{R}_I$, and all $i \in [1 \dots I]$, we have $\rho_i(s|s) > \frac{1}{2} + \epsilon$ for both $s \in \{\pm 1\}$.
[Here, $\rho_i : \mathcal{S} \longrightarrow \Delta(\mathcal{V})$ is projection of ρ onto ith coordinate.]

For any $\rho \in \mathcal{R}_I$, and for both $s \in \{\pm 1\}$ it is clear that $\mathbb{E}[\rho_i(s)] \in \mathcal{C}_s$ for all $i \in [1 \dots I]$. Thus, $F_{\mathrm{maj}}(\mathbb{E}[\rho_i(s)]) = s$, so Identification is satisfied.

Finally, $\mathcal{V}_{\mathrm{maj}}$ is finite, so Minimal Reliability is satisfied.

Thus, if $\mathfrak{R}$ satisfies Asymptotically Weak Average Correlation, then Theorem 1 yields a Condorcet Jury Theorem for correlated, heterogeneous voters (similar to Ladha, 1992).

Let $N \geq 2$, and let $\mathcal{S} := \{1, 2, \ldots N\}$ (a set of N alternatives).

Let $\mathbb{V}_{\rm plu} := \mathbb{R}^N$, and let $\mathcal{V}_{\rm plu}$ and $F_{\rm plu}$ be as in Example 2 above.

Fix $\epsilon > 0$. For all $s \in \mathcal{S}$, let $\mathcal{C}_s := \{\mathbf{r} \in \mathbb{R}^N;\ r_s \geq r_t + \epsilon$ for all $t \neq s\}$.

Let $\mathcal{C}' := \mathcal{C}_1 \sqcup \mathcal{C}_2 \sqcup \cdots \sqcup \mathcal{C}_N$. Then CONTINUITY is satisfied.

Let $\mathfrak{R} = (\mathcal{R}_I)_{I=1}^{\infty}$ be a culture such that, for all $I \in \mathbb{N}$, all $\rho \in \mathcal{R}_I$, and all $i \in [1 \ldots I]$, we have $\rho_i(\mathbf{v}^s|s) > \rho_i(\mathbf{v}^t|s) + \epsilon$, for all $s, t \in \mathcal{S}$ with $s \neq t$.

For all $i \in [1...I]$ and $s \in \mathcal{S}$, we have $\mathbb{E}[\rho_i(s)] = (\rho_i(1|s), ..., \rho_i(N|s)) \in \mathcal{C}_s$. Thus, IDENTIFICATION is satisfied.

Finally, $\mathcal{V}_{\rm plu}$ is finite. So MINIMAL RELIABILITY is satisfied.

Thus, if $\mathfrak{R}$ satisfies ASYMPTOTICALLY WEAK AVERAGE CORRELATION, then Theorem 1 yields a polychotomous CJT: if each voter has some minimal competency, and the voters are only weakly correlated, then the outcome of the *plurality rule* will converge in probability to the correct answer, as the population becomes large.

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Example: The plurality CJT

Let $N \geq 2$, and let $\mathcal{S} := \{1, 2, \ldots N\}$ (a set of N alternatives).

Let $\mathbb{V}_{\rm plu} := \mathbb{R}^N$, and let $\mathcal{V}_{\rm plu}$ and $F_{\rm plu}$ be as in Example 2 above.

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Thus, if $\mathfrak{R}$ satisfies ASYMPTOTICALLY WEAK AVERAGE CORRELATION, then Theorem 1 yields a polychotomous CJT: if each voter has some minimal competency, and the voters are only weakly correlated, then the outcome of the *plurality rule* will converge in probability to the correct answer, as the population becomes large.

Let $N \geq 1$. Let $\mathcal{V} = \mathcal{S}$ be some convex subset of $\mathbb{R}^N$, and let $F_{\rm ave}$ be the averaging rule (i.e. identity function) from Example 3.

CONTINUITY is satisfied (with $\mathcal{C}' := \mathcal{S}$), because $F_{\rm ave}$ is uniformly continuous, and the preimage of each point is a singleton.

Fix $M > 0$. Let $\mathfrak{R} = (\mathcal{R}_I)_{I=1}^{\infty}$ be a culture such that, for all $I \in \mathbb{N}$, $\rho \in \mathcal{R}_I$, and $i \in [1...I]$, we have $\mathbb{E}[\rho_i(\mathbf{s})] = \mathbf{s}$ and $\mathrm{var}[\rho_i(\mathbf{s})] \leq M$, for all $\mathbf{s} \in \mathcal{S}$.

Then IDENTIFICATION and MINIMAL RELIABILITY are satisfied.

Thus, if $\mathfrak{R}$ satisfies ASYMPTOTICALLY WEAK AVERAGE CORRELATION, then Theorem 1 yields the *Wisdom of Crowds* principle for the estimation of some vector-valued quantity: if each voter estimates the quantity, and these estimates are weakly correlated, unbiased, and have finite variance, then the arithmetic mean of the estimates of the voters will converge in probability to the correct answer.

Example: The Wisdom of Crowds

Let $N \geq 1$. Let $\mathcal{V} = \mathcal{S}$ be some convex subset of $\mathbb{R}^N$, and let F_{ave} be the averaging rule (i.e. identity function) from Example 3.

CONTINUITY is satisfied (with $\mathcal{C}' := \mathcal{S}$), because F_{ave} is uniformly continuous, and the preimage of each point is a singleton.

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Then Identification and Minimal Reliability are satisfied.

Thus, if $\mathfrak{R}$ satisfies Asymptotically Weak Average Correlation, then Theorem 1 yields the *Wisdom of Crowds* principle for the estimation of some vector-valued quantity: if each voter estimates the quantity, and these estimates are weakly correlated, unbiased, and have finite variance, then the arithmetic mean of the estimates of the voters will converge in probability to the correct answer.

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Then $\textsc{Identification}$ and $\textsc{Minimal Reliability}$ are satisfied.

Thus, if $\mathfrak{R}$ satisfies $\textsc{Asymptotically Weak Average Correlation}$, then Theorem 1 yields the *Wisdom of Crowds* principle for the estimation of some vector-valued quantity: if each voter estimates the quantity, and these estimates are weakly correlated, unbiased, and have finite variance, then the arithmetic mean of the estimates of the voters will converge in probability to the correct answer.

Let $\mathcal{S}$ be a finite set. Let $p : \mathcal{S} \longrightarrow \Delta(\mathcal{S})$ be a function.

Idea: For any $s, t \in \mathcal{S}$, $p(t|s)$ is the probability that a voter *believes* the true state is t, when it is actually s. (Call this the *error model*.)

Let $\mathbb{V} := \mathbb{R}^{\mathcal{S}}$. For all $r \in \mathcal{S}$, define $\mathbf{v}^r := (v_s^r)_{s \in \mathcal{S}} \in \mathbb{V}$ by setting $v_s^r := \log[p(r|s)]$, for all $s \in \mathcal{S}$.

Let $\mathcal{V} := \{\mathbf{v}^r\}_{r \in \mathcal{S}}$. Let $\mathcal{C}$ be the convex hull of $\mathcal{V}$.

Let $F_{\log}^p := F_{\mathrm{scr}} : \mathcal{C} \longrightarrow \mathcal{S}$ be the corresponding scoring rule (as in Example 5).

We will refer to this as a *log-likelihood* scoring rule.

Application: Suppose the voters are independent random variables. If the error model has "enough symmetry", then the outcome of $F_{\log}^p$ will be the *maximum likelihood estimator* (MLE) of the true state of nature. Conversely, *any* scoring rule is a log-likelihood rule for some error model. In many cases, these are actually MLEs (Pivato, SC&W, 2011, Thrm 2.2(b)).

Example. (a) The *Kemeny rule* is the MLE for a natural error model on the space of preference orders (Young, 1986, 1988, 1995, 1997).

(b) The generalized median rule is the MLE for any *exponential* error model on any "homogeneous" metric space (Pivato, SC&W, 2011, Corollary 3.2).

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Any error model p_i defines a function $\rho_i : \mathcal{S} \longrightarrow \Delta(\mathcal{V})$ in the obvious way.

Fix $\delta > 0$. Let $\mathfrak{R} = (\mathcal{R}_I)_{I=1}^{\infty}$ be a culture such that, for all $I \in \mathbb{N}$, all $\rho \in \mathcal{R}_I$, and all $i \in [1 \ldots I]$, ρ_i corresponds to an error model p_i such that $|p_i(t|s) - p(t|s)| < \delta$ for all $t, s \in \mathcal{S}$.

Suppose $\mathfrak{R}$ satisfies Asymptotically Weak Average Correlation. If δ is small enough, then Theorem 1 implies: If a large population of voters is drawn from the culture $\mathfrak{R}$, then the log-likelihood rule $F_{\log}^{p}$ will identify the true state of nature, with probability arbitrarily close to 1.

Example: Let $\mathcal{S}$ be the space of preference orders on some set of alternatives. Then this conclusion holds for the Kemeny rule, given the error model proposed by Young (1986-97).

Proof sketch: If $p(t|s) > 0$ for all $t, s \in \mathcal{S}$, then $F_{\log}^{p}$ satisfies Minimal Reliability with respect to any populace $\mathcal{P}$.

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Any error model p_i defines a function $\rho_i : \mathcal{S} \longrightarrow \Delta(\mathcal{V})$ in the obvious way. Fix $\delta > 0$. Let $\mathfrak{R} = (\mathcal{R}_I)_{I=1}^{\infty}$ be a culture such that, for all $I \in \mathbb{N}$, all $\rho \in \mathcal{R}_I$, and all $i \in [1 \ldots I]$, ρ_i corresponds to an error model p_i such that $|p_i(t|s) - p(t|s)| < \delta$ for all $t, s \in \mathcal{S}$.

Suppose $\mathfrak{R}$ satisfies ASYMPTOTICALLY WEAK AVERAGE CORRELATION. If δ is small enough, then Theorem 1 implies: If a large population of voters is drawn from the culture $\mathfrak{R}$, then the log-likelihood rule $F^p_{\log}$ will identify the true state of nature, with probability arbitrarily close to 1.

Example: Let $\mathcal{S}$ be the space of preference orders on some set of alternatives. Then this conclusion holds for the Kemeny rule, given the error model proposed by Young (1986-97).

Proof sketch: If $p(t|s) > 0$ for all $t, s \in \mathcal{S}$, then $F^p_{\log}$ satisfies MINIMAL RELIABILITY with respect to any populace $\mathcal{P}$.

Fix $\epsilon > 0$. For all $s \in \mathcal{S}$, let $\mathcal{C}^{\epsilon}_s := \{\mathbf{c} \in \mathcal{C};\ c_s \geq c_t + \epsilon$ for all $t \neq s\}$.

If $\mathcal{C}'_{\epsilon} := \bigcup_{s \in \mathcal{S}} \mathcal{C}^{\epsilon}_s$, then $F^p_{\log}$ satisfies CONTINUITY when restricted to $\mathcal{C}'_{\epsilon}$.

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Suppose $\mathfrak{R}$ satisfies Asymptotically Weak Average Correlation. If δ is small enough, then Theorem 1 implies: If a large population of voters is drawn from the culture $\mathfrak{R}$, then the log-likelihood rule $F_{\log}^{p}$ will identify the true state of nature, with probability arbitrarily close to 1.

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In all these examples, the culture $\mathfrak{R}$ satisfied CONTINUITY and IDENTIFICATION with respect to some subset $\mathcal{C}' \subseteq \mathcal{C}$.

If $\mathfrak{R}$ does *not* satisfy these two conditions, then epistemically useful social choice is probably impossible —the voters are just too foolish.

Thus, the key conditions are MINIMAL AVERAGE RELIABILITY and ASYMPTOTICALLY WEAK AVERAGE CORRELATION, which are both conditions on the *correlation structure* of the culture.

Definition. A correlation structure $\mathfrak{B}$ is *sophogenic* if it satisfies MINIMAL AVE. RELIABILITY and ASYMPT. WEAK AVERAGE CORRELATION.

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Part III

Social networks

Let $\mathcal{I} := [1 \ldots I]$. Let "$\sim$" be a *graph* (symmetric, reflexive binary relation) on $\mathcal{I}$.

Idea: If $i \sim j$, then i and j are somehow "socially connected" (e.g. friends, family, colleagues, etc.).

We will refer to $\sim$ as a *social network*.

Problem. We cannot assume that we know the exact topology of the social network. We can only know certain broad qualitative properties....

Definition. A *social web* is a sequence $\mathfrak{N} = (\mathcal{N}_I)_{I=1}^{\infty}$, where, for all $I \in \mathbb{N}$, $\mathcal{N}_I$ is a set of possible social networks of size I.

Let $\sim$ be some network. For any $i \in \mathcal{I}$, define $\deg_\sim(i) := \#\{j \in \mathcal{I};\ i \sim j\}$.

The *degree distribution* of the network "$\sim$" is the probability distribution $\mu_\sim \in \Delta(\mathbb{N})$ such that, for all $d \in \mathbb{N}$,

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$$\lim_{I\to\infty} \sup \{ D(\mu_\sim) \; ; \; \sim \in \mathcal{N}_I \} \;=\; 0,$$

where $D(\mu_\sim) \;:=\; \sum_{n=1}^{\infty} n \cdot |\mu_\sim(n) - \mu_*(n)|.$

Example. Empirically, many social networks have a *power law* distribution:

$$\mu_*(d) \;=\; \frac{K}{d^\alpha}, \qquad \text{for all } d \in \mathbb{N}.$$

where $\alpha > 1$ and $K := \left(\sum_{d=1}^{\infty} d^\alpha\right)^{-1}$. (Typically, $2 < \alpha < 3$.)

- For any $I \in \mathbb{N}$ and $\mathbf{B} \in \mathcal{B}_I$, there is some $\sim$ in $\mathcal{N}_I$ such that, for all $i, j \in [1 \dots I]$, we have $b_{i,j} \neq 0$ only if $i \sim j$.
- There is some $M > 0$ such that, for any $I \in \mathbb{N}$ and $\mathbf{B} \in \mathcal{B}_I$, we have $|b_{i,j}| < M$ for all $i, j \in [1 \dots I]$.

Asymptotic degree distributions

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Definition. Let $(\mathcal{B}_I)_{I=1}^{\infty}$ be a correlation structure. Let $(\mathcal{N}_I)_{I=1}^{\infty}$ be a social web. Say $(\mathcal{B}_I)_{I=1}^{\infty}$ is *subordinate* to $(\mathcal{N}_I)_{I=1}^{\infty}$ if:

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Proposition 2. ("Social network cultures can be sophogenic")
Suppose a social web $\mathfrak{N}$ has a well-defined asymptotic degree distribution as $I \to \infty$. Then any correlation structure subordinate to $\mathfrak{N}$ is sophogenic.
(So Theorem 1 applies to such cultures.)

Remark: Proposition 2 does *not* say that "all cultures arising from social networks are sophogenic". It may be that the social web $\mathfrak{N}$ has *no* asymptotic degree distribution as $I \to \infty$.

Nonexamples. Erdös-Renyi random graphs, "star" networks....
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In these cases, there *is no* asymptotic degree distribution as $I\to\infty$. So Proposition 2 does not apply

However, Proposition 2 *does* apply to the sort of social network topologies which arise in practice (e.g. power law degree distributions).

Proposition 2. ("Social network cultures can be sophogenic")
Suppose a social web $\mathfrak{N}$ has a well-defined asymptotic degree distribution as $I \to \infty$. Then any correlation structure subordinate to $\mathfrak{N}$ is sophogenic. (So Theorem 1 applies to such cultures.)

Remark: Proposition 2 does *not* say that "all cultures arising from social networks are sophogenic". It may be that the social web $\mathfrak{N}$ has *no* asymptotic degree distribution as $I \to \infty$.

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However, Proposition 2 *does* apply to the sort of social network topologies which arise in practice (e.g. power law degree distributions).

Part IV

DeGroot models of social influence

Let $\mathbf{G} := [g_{i,j}]_{i,j\in\mathcal{I}}$ be an $I \times I$ *stochastic matrix*.

That is, $g_{i,j} \geq 0$ for all $i, j \in \mathcal{I}$, and for all $i \in \mathcal{I}$, $\sum_{j\in\mathcal{I}} g_{i,j} = 1$.

Idea: $g_{i,j}$ is the "influence" of individual j on individual i (DeGroot, 1974).

For any $j \in \mathcal{I}$, let $\overline{g}_j := \frac{1}{I}\sum_{i\in\mathcal{I}} g_{i,j} =$ ["average influence" of individual j].

Define the *demagoguery index* of $\mathbf{G}$ by $\delta(\mathbf{G}) := \max\{\overline{g}_j\}_{j\in\mathcal{I}}$.

Issue. We do not know $\mathbf{G}$. We only know that it comes from some family...

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Dialogues and demagogues

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By combining a dialogue with an uncorrelated culture, we will construct a correlated culture...

Suppose $\mathcal{V}$ is a *convex* subset of a vector space $\mathbb{V}$.
Let $\mathbf{V} = (\mathbf{v}_i)_{i=1}^I \in \mathcal{V}^I$. Let $\mathbf{G}$ be an $I \times I$ stochastic ("influence") matrix.

Define $\mathbf{G} \cdot \mathbf{V} := (\mathbf{v}'_i)_{i=1}^I$, where, for all $i \in \mathcal{I}$, $\mathbf{v}'_i := \sum_{j=1}^{I} g_{i,j}\, \mathbf{v}_j$.

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Let $\rho : \mathcal{S} \longrightarrow \Delta(\mathcal{V}^I)$ be an *uncorrelated* CBM. Fix $s \in \mathcal{S}$. Let $\mathbf{V} = (\mathbf{v}_i)_{i=1}^I$ be a $\rho(s)$-random profile. Then $\mathbf{G} \cdot \mathbf{V}$ is a (*correlated*) random profile.

Let $\mathbf{G} \odot \rho(s)$ denote the probability distribution of $\mathbf{G} \cdot \mathbf{V}$.

If we do this for all $s \in \mathcal{S}$, then we obtain a (*correlated*) collective behaviour model, which we denote by $\mathbf{G} \odot \rho : \mathcal{S} \longrightarrow \Delta(\mathcal{V}^I)$.

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From dialogue to culture

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From dialogue to culture

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Other applications: Probability aggregation.

Upshot: Large populations have a high probability of finding the truth.

This result survives correlations between voters, as long as these correlations are "not too strong". Concrete examples include:

- Social networks with asymptotic degree distributions.
- DeGroot models of social influence which are "nondemagoguic".

Other models:

- DeGroot model combined with "meritocratic institutions", where smarter voters get more influence.
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Other results:

- Tradeoff between group size and average voter competency.

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- Culture having "subcultures" with strong internal correlations (bad).

Other results:

- Tradeoff between group size and average voter competency.

Thank you.