Distance Rationalizability: Information Merging through Consensus Seeking

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EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

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Coding Theory: Error Correcting Codes

Original string
010

Encoded string
000111000

Received string
010011100

Decoded string
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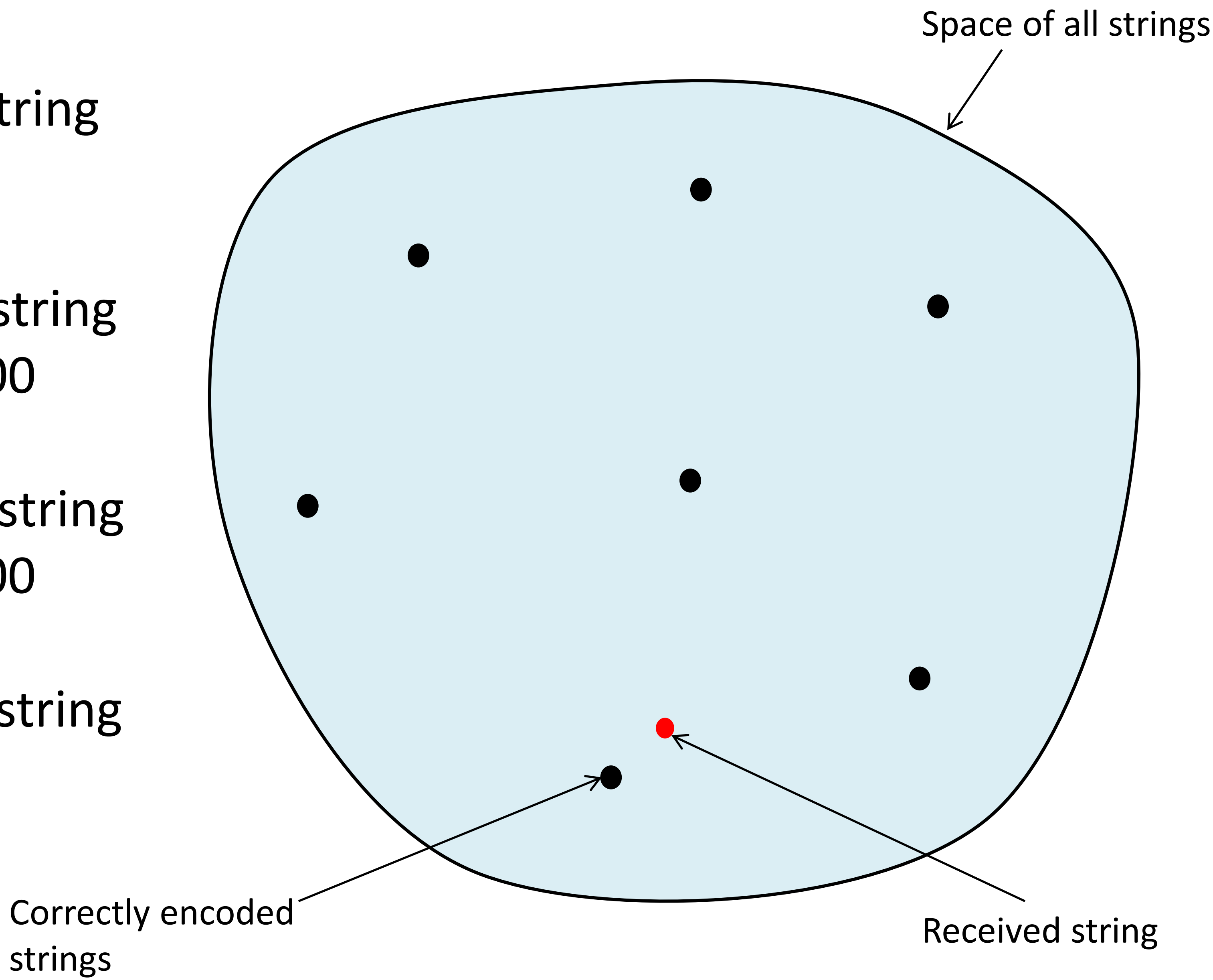

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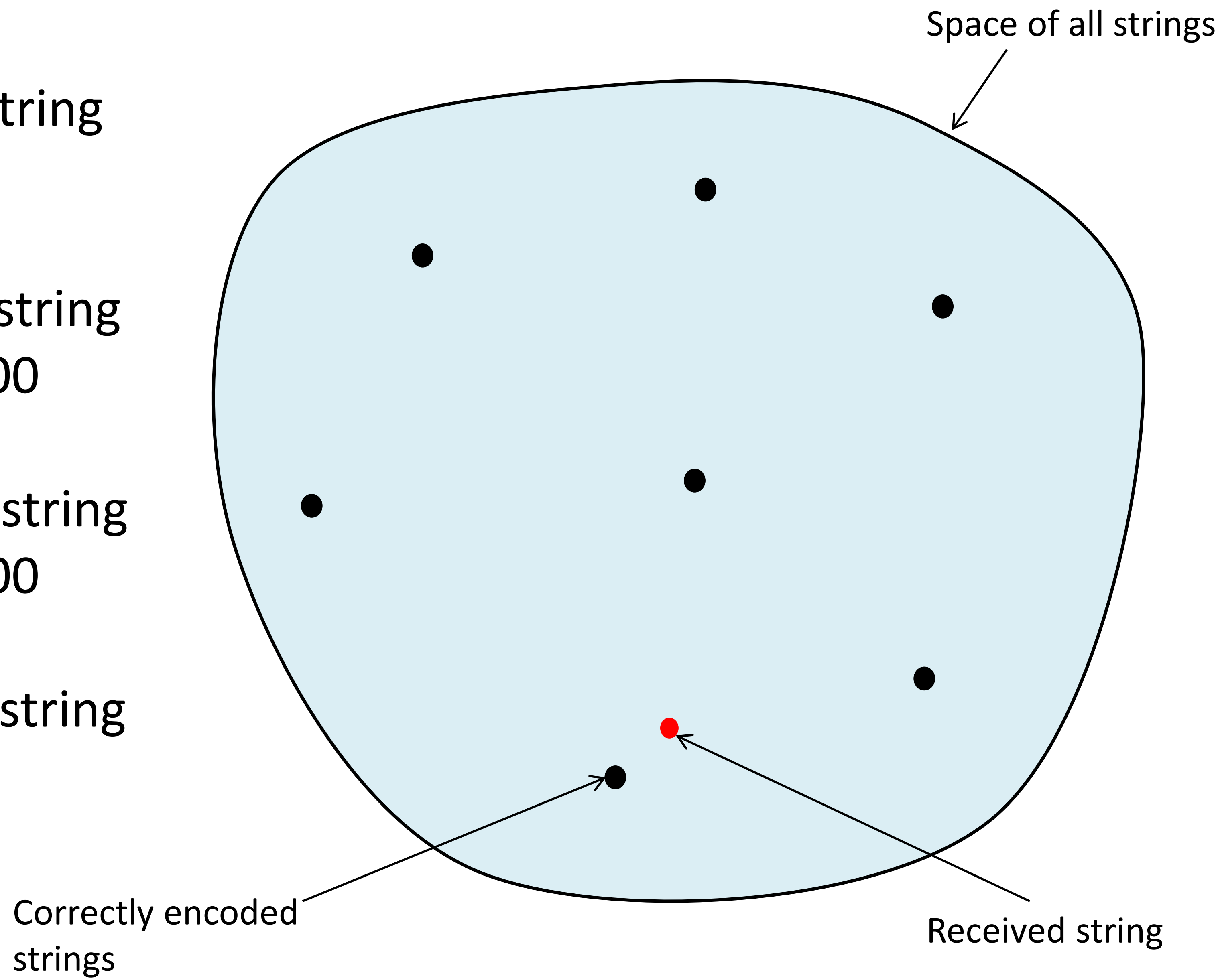
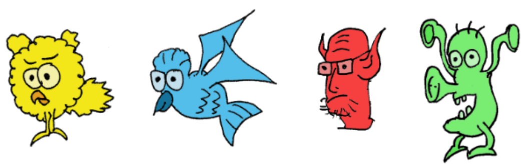


Distance Rationalizability

Space:

All elections over



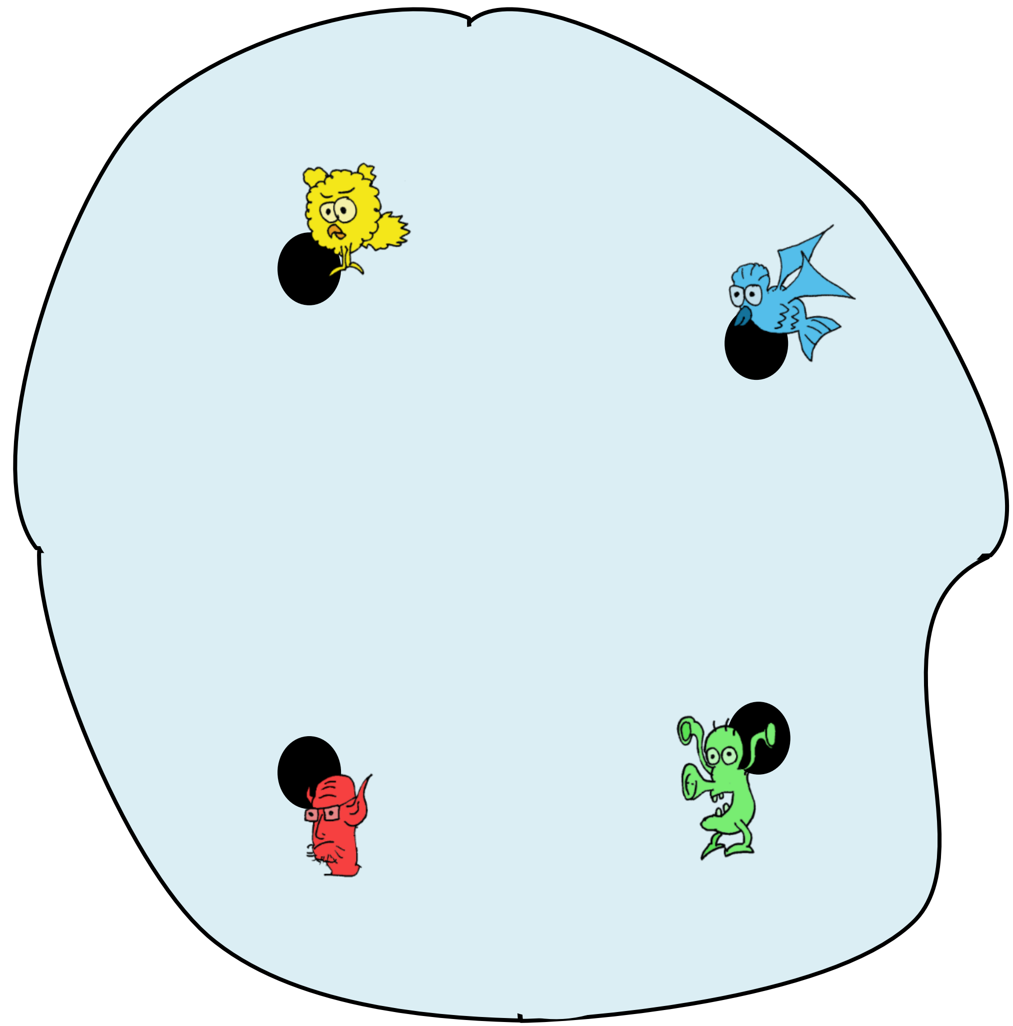
Elections with clear winners

(consensus notion)

- Condorcet winner?
- Always ranked first?
- Identical preference orders?

Distance notion

- Swap distance? Hamming distance?



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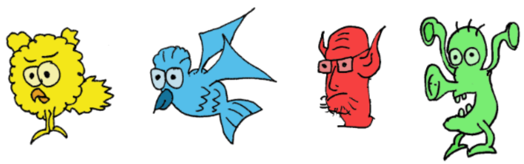

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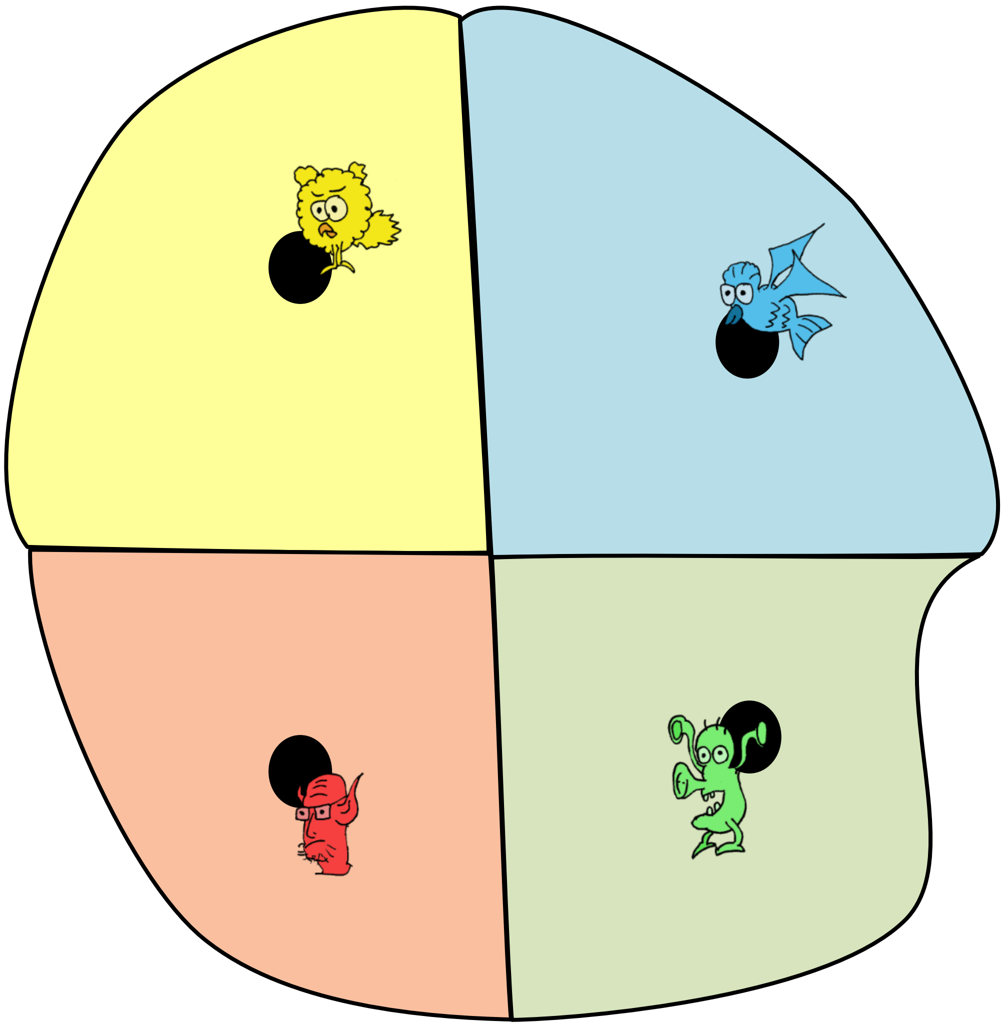

Two (?) Main Components

Consensus notions	Distances between elections

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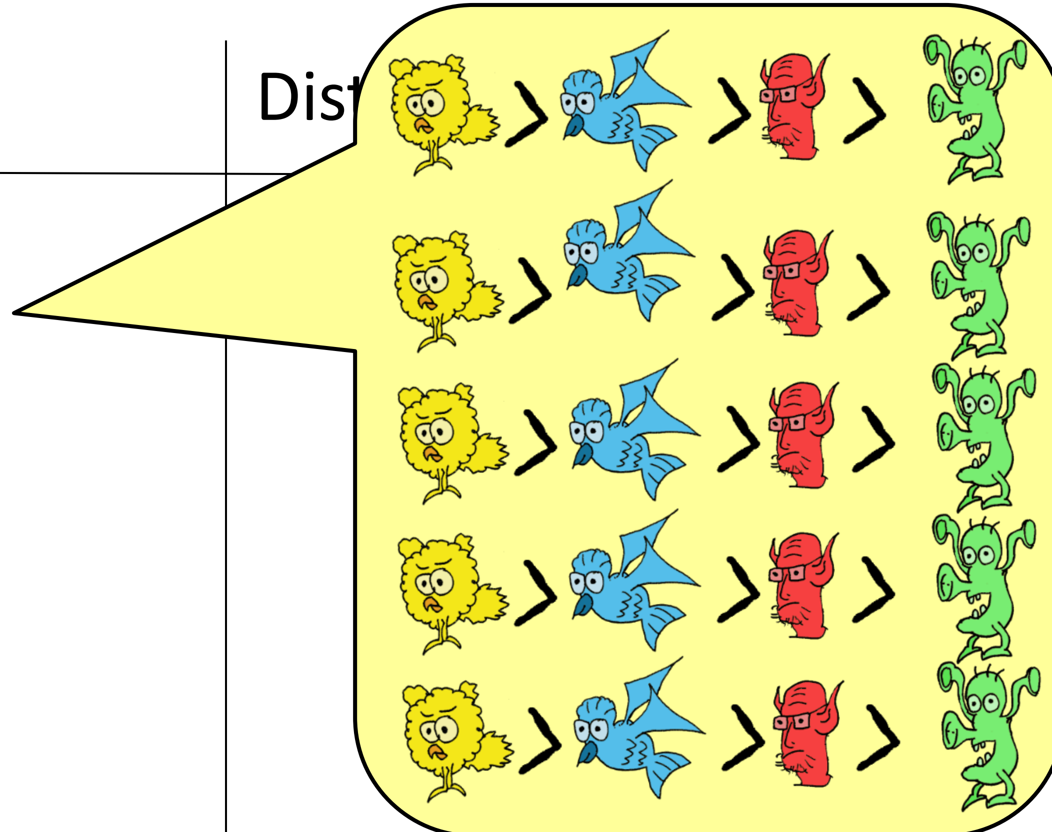
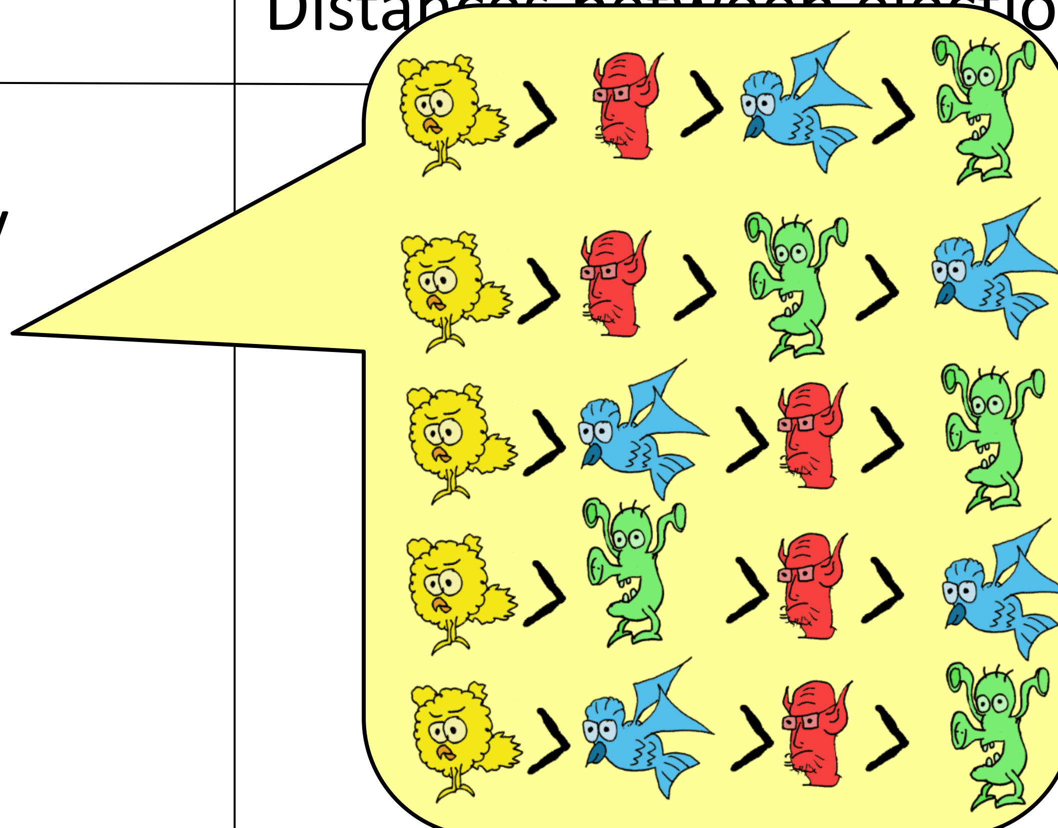
Consensus notions	Dist ns
S – strong unanimity	



Two (?) Main Components

Consensus notions	Distances between elections
S – strong unanimity	
U – weak unanimity	



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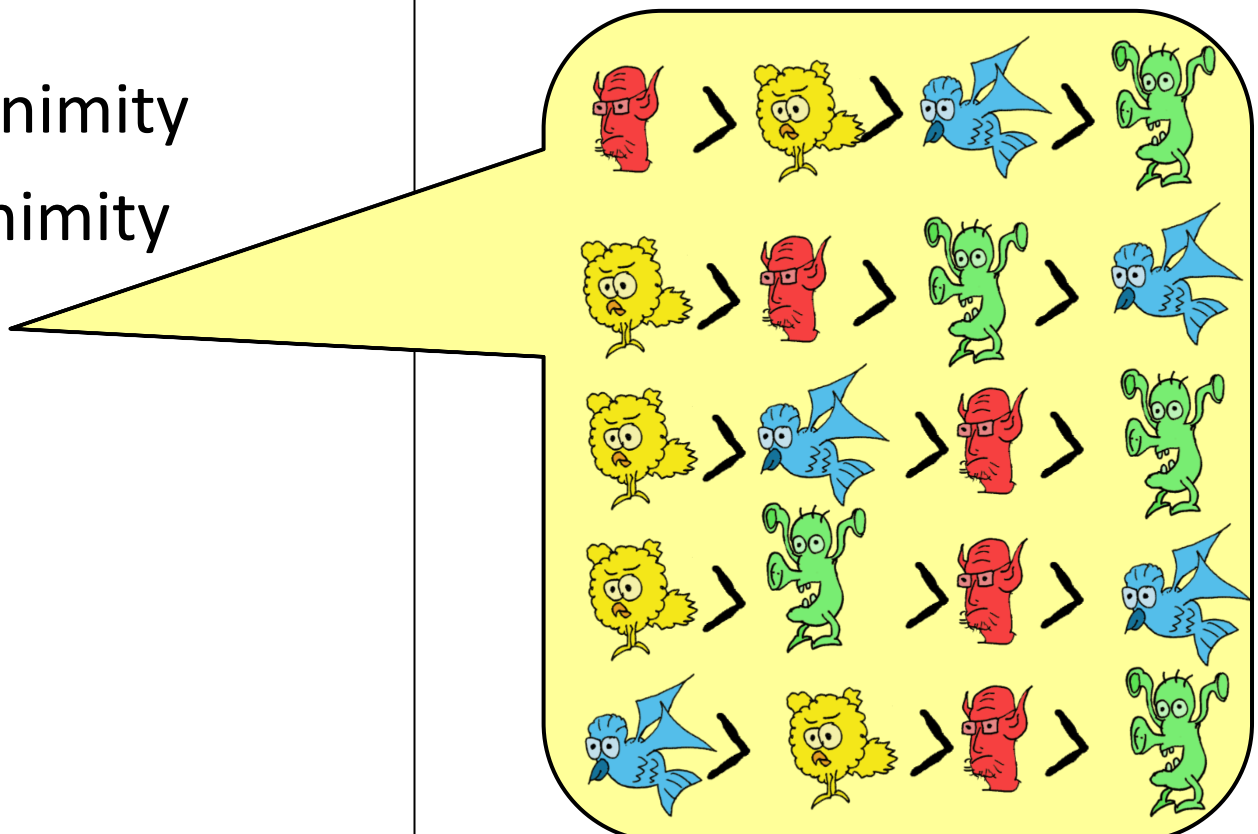
S – strong unanimity U – weak unanimity M – majority	

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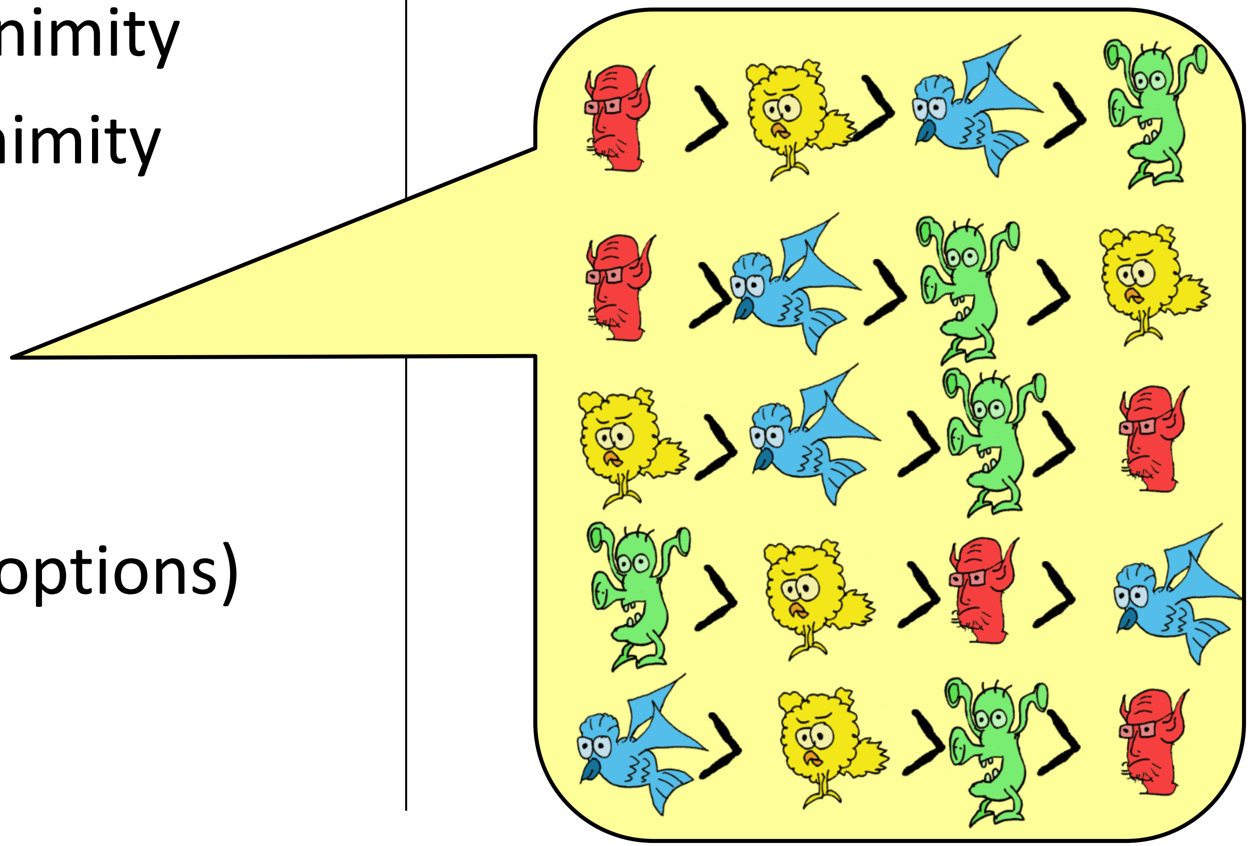
S – strong unanimity U – weak unanimity M – majority C – Condorcet … (lot's of other options)	

Two (?) Main Components

Consensus notions	Distances between elections
S – strong unanimity	Hamming distance (d_{Ham})
U – weak unanimity	Swap distance (d_{Swap})
M – majority	…
C – Condorcet	Random whatever ☺
…	
(lot's of other options)	

Two (?) Main Components

Co … ions

S

U

M

C

…

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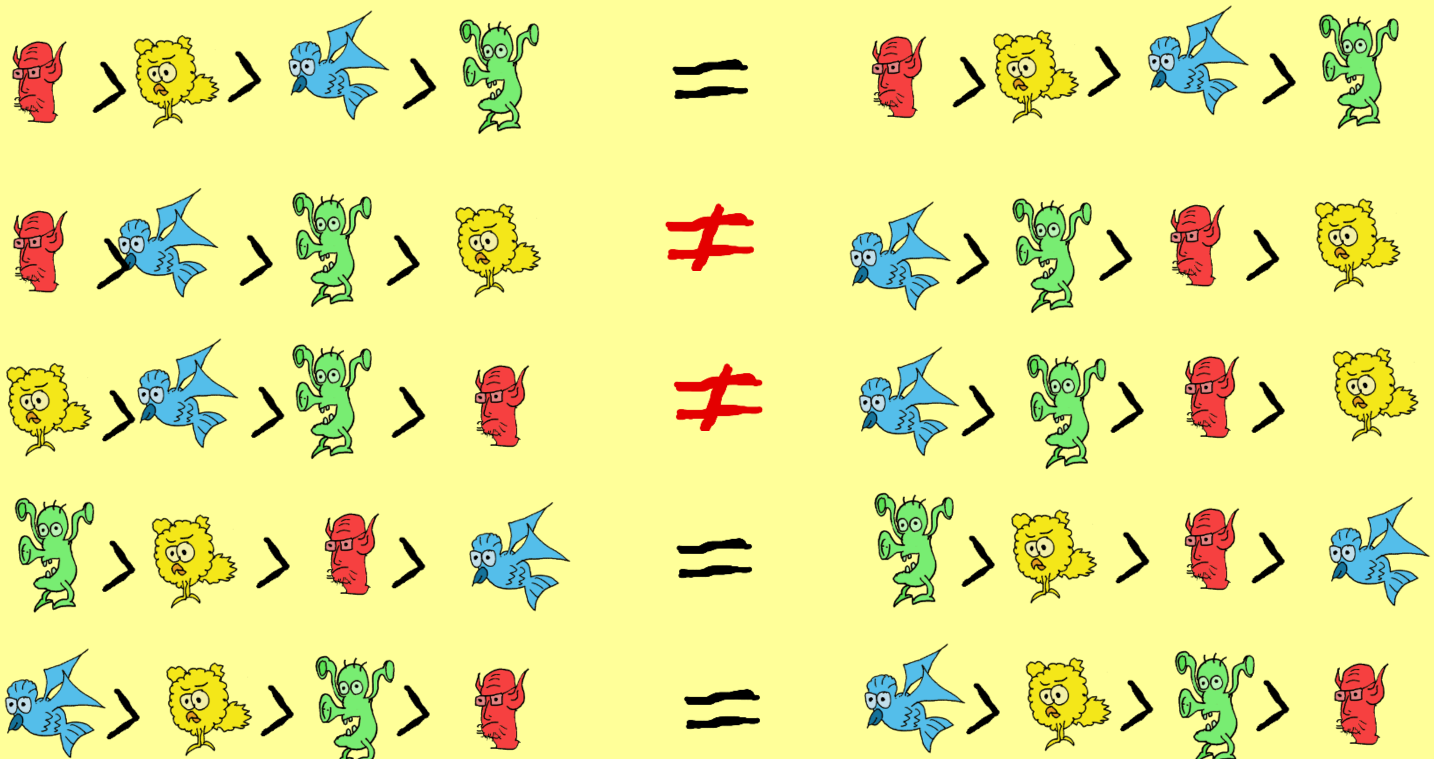
Hamming distance



Distance = 2 (two votes differ)

Two (?) Main Components

Consensus notions	Distances between elections
S – strong unanimity	Hamming distance (d_{Ham})
U – weak unanimity	Swap distance (d_{Swap})
M – majority	…
C – Condorcet	Random whatever ☺
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Two (?) Main Components

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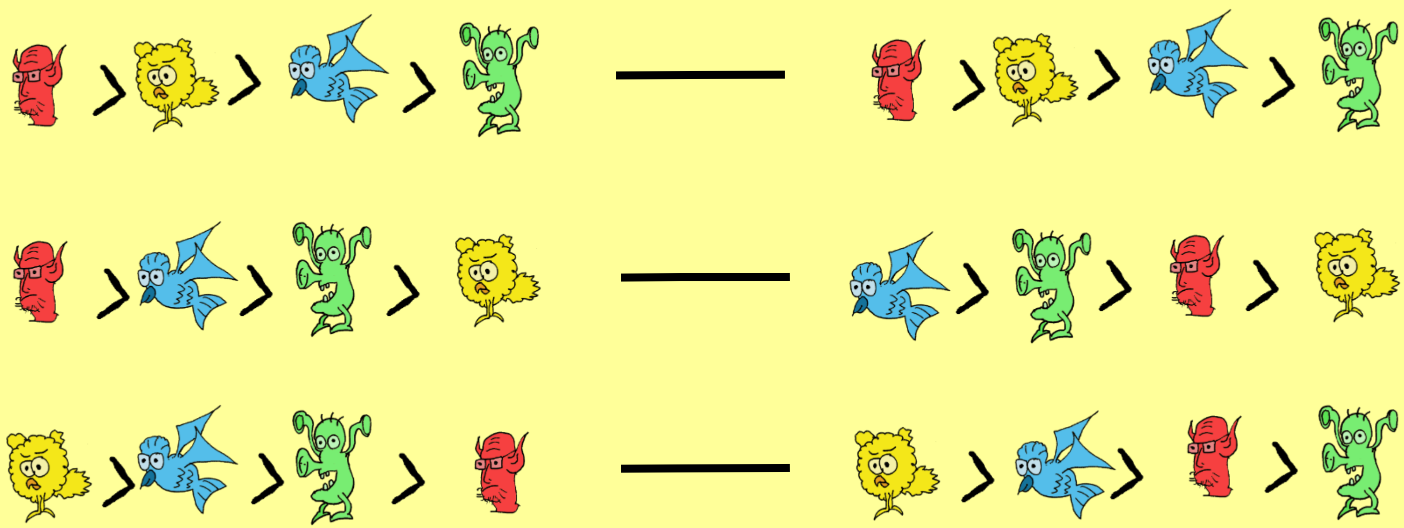
Swap distance (Kendal Tau, bubble sort, ...)



Distance = 3 (three adjacent swaps)

Putting Together a Voting Rule

Setting

K – consensus notion (S, U, M, C, …)

d – distance among elections

R = (K, d) – a voting rule

Given election E = (C, V)

C – set of candidates

V – profile of preference orders

R = (K, d) selects candidate c such that the consensus from K where c wins is d-closest to V

Natural Rules Fit the Framework

(U, d_{Ham}) = Plurality

(U, d_{Swap}) = Borda

(C, d_{Swap}) = Dodgson

(S, d_{Swap}) = Kemeny

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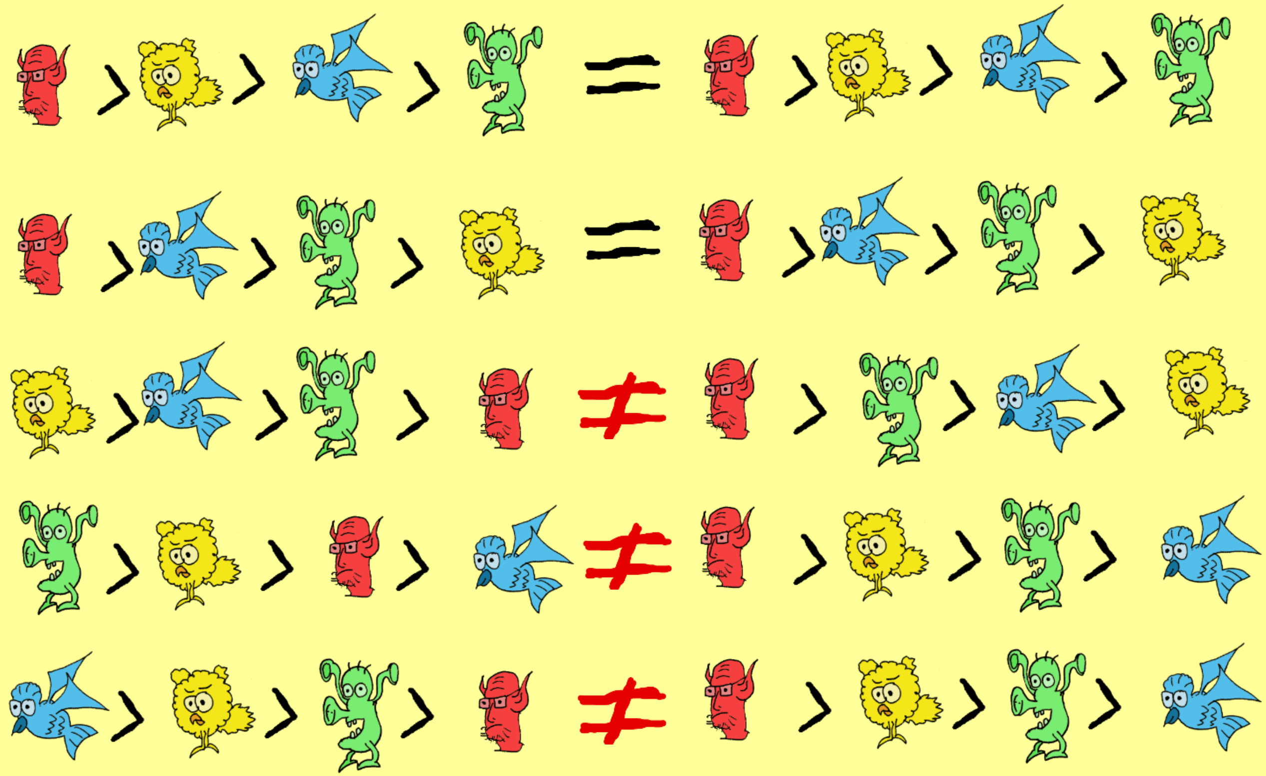

Distance = |V| - plurality-score = 5-2 = 3

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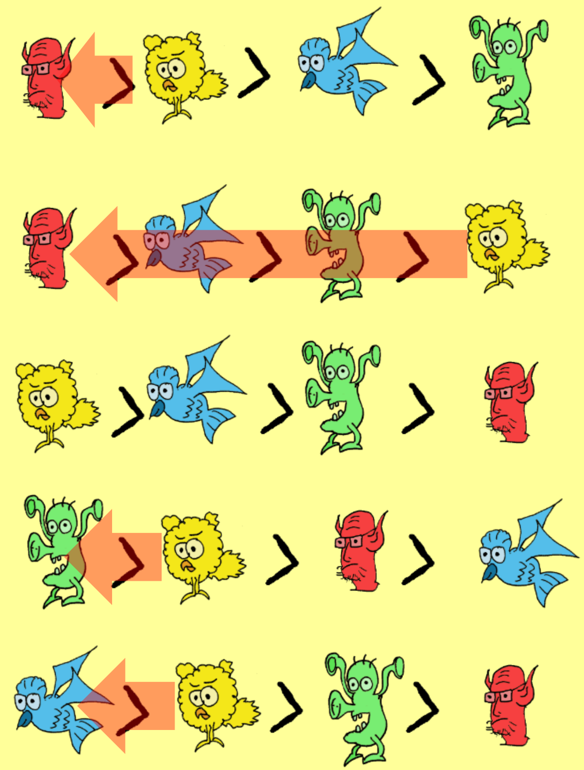

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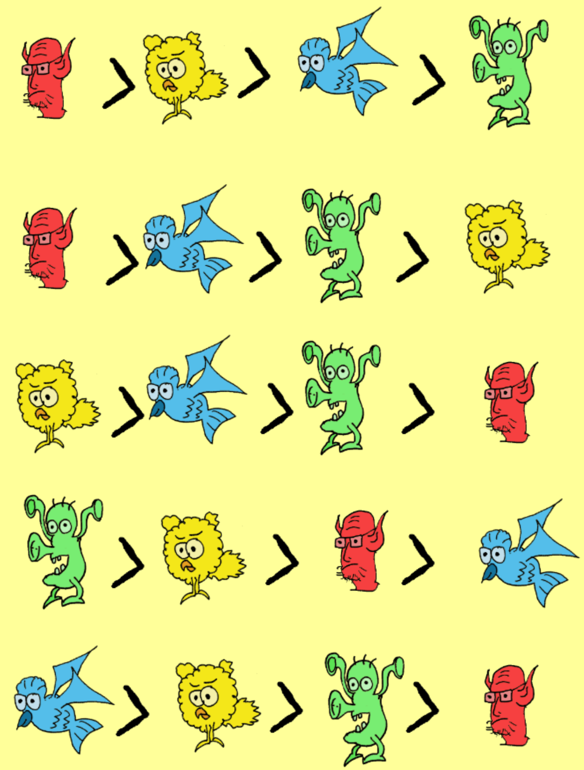


Distance = K – Borda-score

Natural Rules Fit the Framework

(U, d_{Ham}) = Plurality

(U, d_{Swap}) = Borda

(C, d_{Swap}) = Dodgson

(S, d_{Swap}) = Kemeny

(C, d_{Swap}) = Dodgson

By definition – Dodgson's rule picks the candidate who can become Condorcet winner by fewest swaps

Natural Rules Fit the Framework

(U, d_{Ham}) = Plurality

(U, d_{Swap}) = Borda

(C, d_{Swap}) = Dodgson

(S, d_{Swap}) = Kemeny

(S, d_{Swap}) = Kemeny

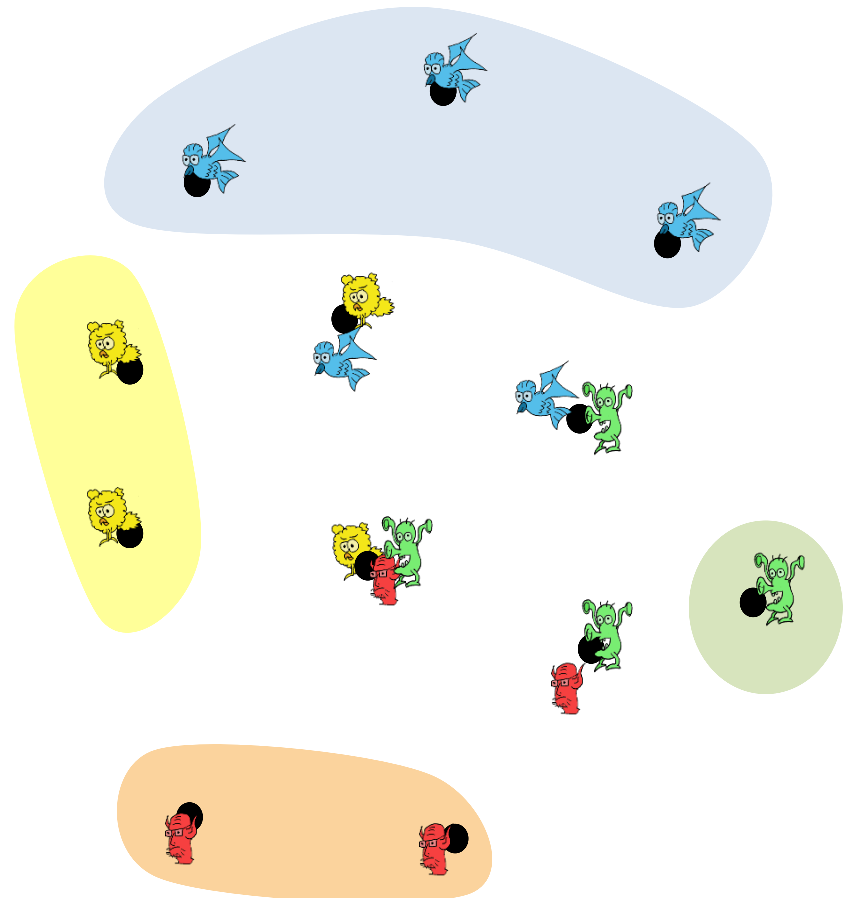
By definition – the consensus ranking is the Kemeny ranking, and we want to reach with fewest total number of swaps

Weird Rules Fit the Framework („All” of them)

Thm. For (almost) every voting rule R there is a consensus class K and a distance function d such that:

$$R = (K, d)$$

Typically, K can be the strong unanimity (S)



All elections under consideration are vertices

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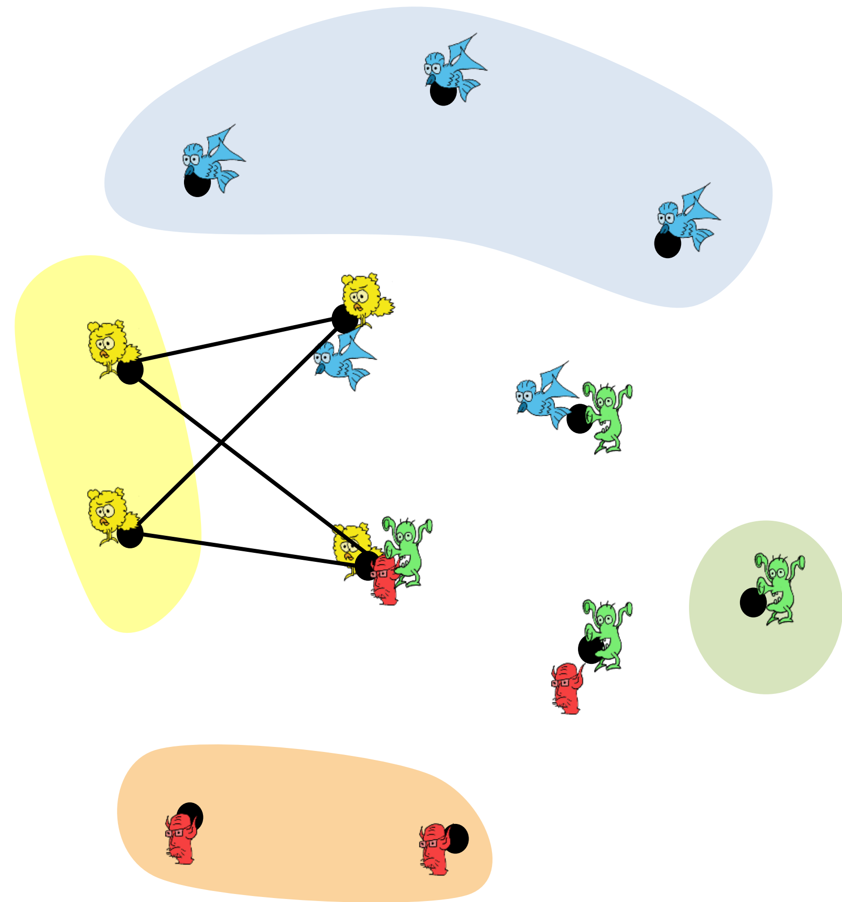

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Weird Rules Fit the Framework („All” of them)

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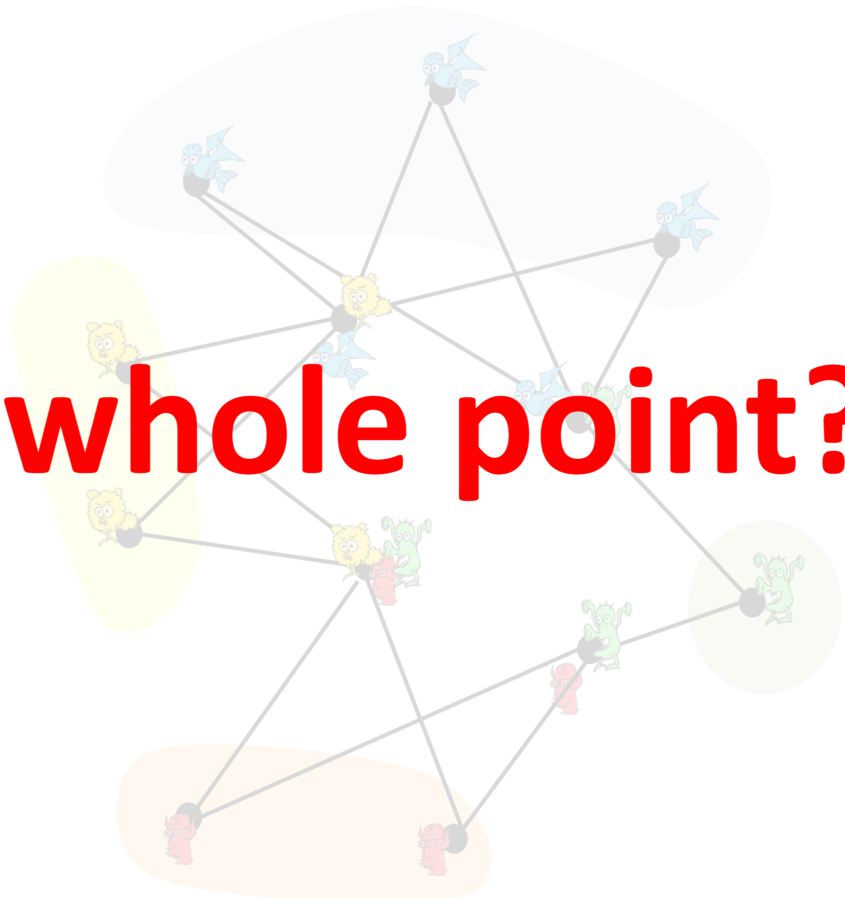
So what’s the whole point?

Typically, K can be the strong unanimity (S)



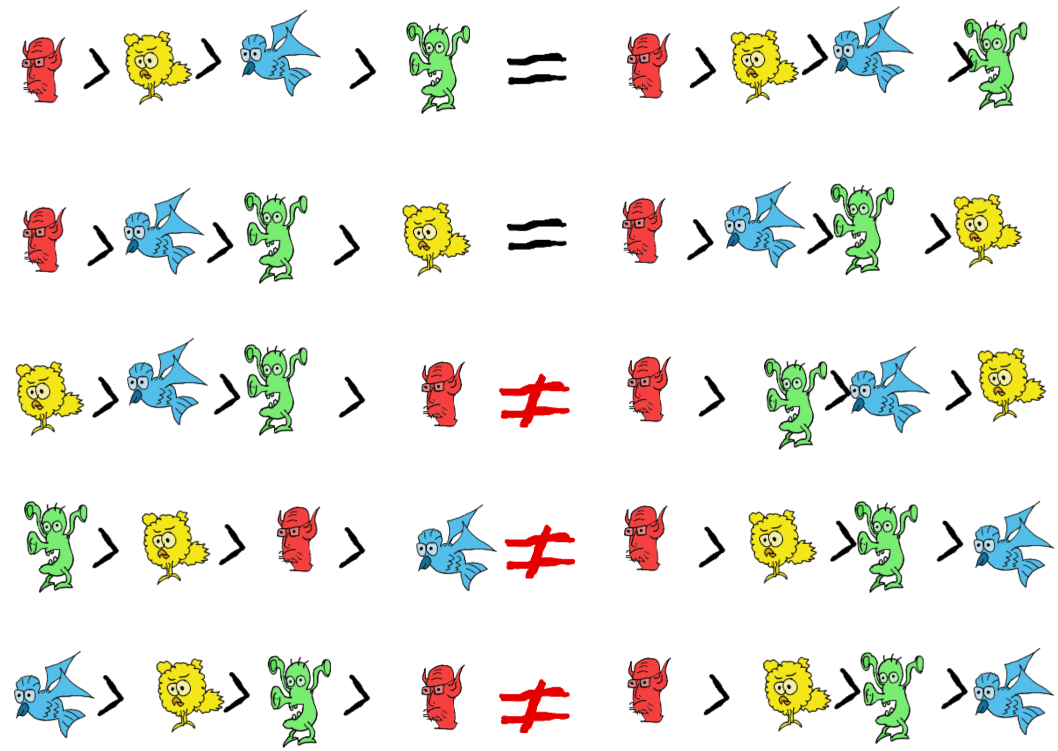
All elections under considerations are vertices
We use the shortest-path distance

Good Distance Rationalizations Are Essential

Some distances and consensus notions are clearly more natural than others.

Consensus:
Stick to S, U, M, and C

Distances:
Consider Plurality or Borda…



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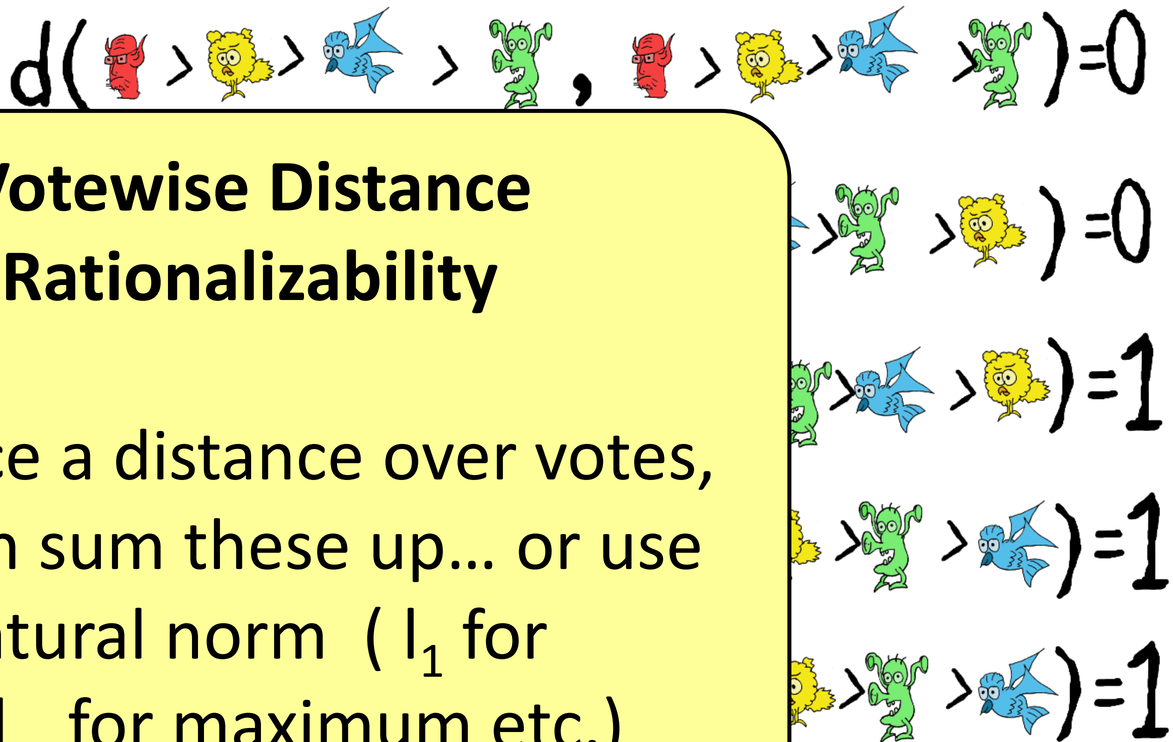


Votewise Distance Rationalizability

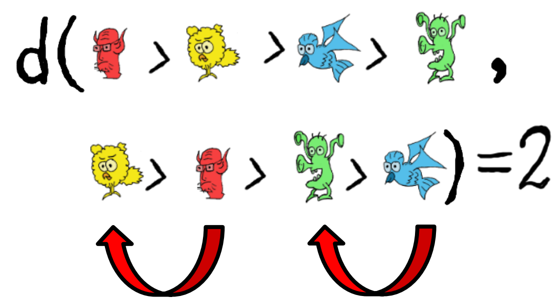
Introduce a distance over votes, and then sum these up… or use some natural norm (l_1 for adding, l_∞ for maximum etc.)

Examples of Natural Distances

Discrete distance

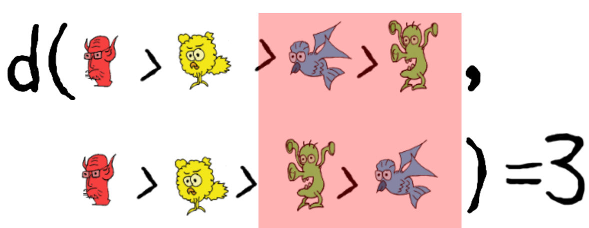
Are the votes the same or not?

Swap distance



How many swaps of adjacent candidates are needed to transform one into the other?

Sertel distance



At which position votes differ?

Voting rule	Consensus class	Distance over votes	Norm
Plurality	$\mathcal{U}$	d_{discr}	ℓ_1
Plurality	$\mathcal{M}$	d_{discr}	ℓ_1
Plurality	$\mathcal{S}$	no name	ℓ_1
Voter replacement rule	$\mathcal{C}$	d_{discr}	ℓ_1
Kemeny	$\mathcal{S}$	d_{swap}	ℓ_1
Borda	$\mathcal{U}$	d_{swap}	ℓ_1
Threshold	$\mathcal{U}$	d_{swap}	ℓ_∞
$\mathcal{M}$-Borda	$\mathcal{M}$	d_{swap}	ℓ_1
Dodgson	$\mathcal{C}$	d_{swap}	ℓ_1
Dodgson$^\infty$	$\mathcal{C}$	d_{swap}	ℓ_∞
Borda	$\mathcal{U}$	d_{spear}	ℓ_1
Borda	$\mathcal{U}$	d_{sert}	ℓ_1
scoring rule $\mathcal{R}_\alpha$	$\mathcal{U}$	$d_{\alpha\text{-swap}}$	ℓ_1
scoring rule $\mathcal{R}_\alpha$	$\mathcal{U}$	d_α	ℓ_1
$\mathcal{M}$-scoring rule $\mathcal{M}$-$\mathcal{R}_\alpha$	$\mathcal{M}$	d_α	ℓ_1
Simplified Bucklin	$\mathcal{M}$	$d_{\infty\text{-spear}}$	ℓ_∞
Simplified Bucklin	$\mathcal{M}$	d_{sert}	ℓ_∞
Litvak	$\mathcal{S}$	d_{spear}	ℓ_1

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Connection to MLE framework

Distance rationalization with respect to strong unanimity (S) implies noise model for MLE approach (and the other way round for a natural family of noise models) ← **needs some caution!!!**

Axiomatic Properties and Distance Rationalizability

Anonymity and neutrality

Derived directly from the distance over prefernece orders and the aggregating norm.

Monotonicity

Not completely trivial!

Possible to derive monotonicity of a votewise DR rule from the properties of the distance and the norm

Rank monotonicity *of a distance: A vote where b is ahead of c is closer to a vote that ranks b on top than to one that ranks c on top*

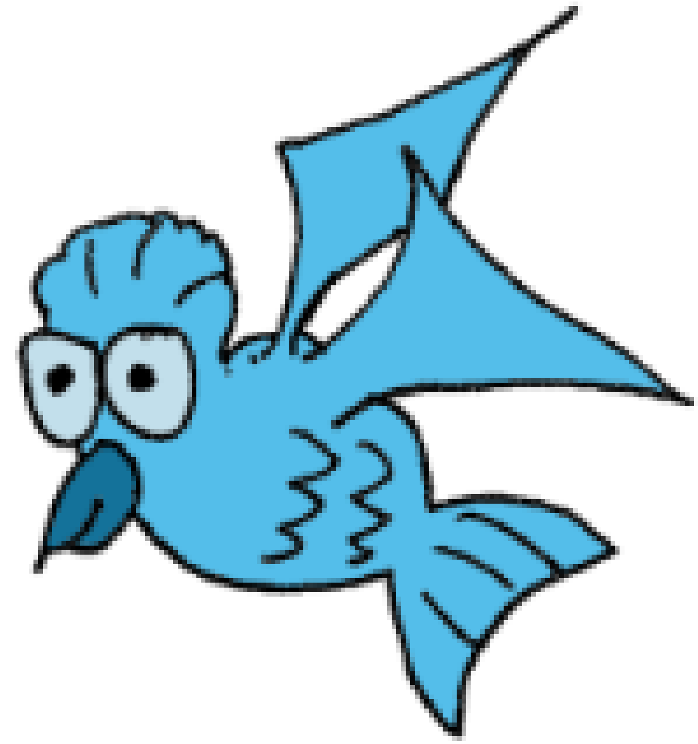
Continuity, Homogeneity, Consistency

Continuity – add enough elections with a given winner and the result will be as they want → **satisfied by votewise DR rules for S and U**

Homogeneity – clone each voter the same number of times, and the result does not change → **satisfied by votewise DR for S and U under l_1, and for almost all votewise DR for l_∞**

Consistency – satisfied by scoring rules → **DR-based characterization of scoring rules**

Conclusions



- Distance rationalizability
  - Very general framework
  - Generates new rules easily
  - Provides insights into new rules
- Possible extensions?
  - Other objects to aggregate (tournaments? Partial orders?)
  - Theoretical justification for consensus notions?

Thanks!