

# Schedule ICATA 2024

**Location:**

Faculty of Medicine

Str. Lucian Blaga, No.2A, Sibiu

Senate Hall

**Thursday, July 18**

**08:00-9:30** Registration

**09:30-09:50** Opening Ceremony

**Plenary Lecture**

**Chairman: Ioan Raşa**

**09:50-10:30** **Gianluca Vinti**, *Discrete and semi-discrete sampling type operators and their applications to image segmentation problems*

**Lecture**

**Chairman: Gianluca Vinti**

**10:30-10:50** **Rosario Corso**, *Mean sampling Kantorovich operators*

**10:50-11:10** **Arianna Travaglini, Gianluca Vinti**, *Non-linear sampling Durrmeyer operator and applications*

**11:10-11:40** Coffee break

## Lecture

Chairman: Gabriel Prajitura

**11:40-12:00** Maryam Mohammadi, Mohammad Heidari, *RBFs as surfaces of revolution*

**12:00-12:20** **Dorin Dumitrascu**, *On the non-holonomic character of first time returns in the Euclidean lattice*

**12:20-12:40** **Valentin Gabriel Cristea**, *New estimates for the Wallis' ratio* (online)

**12:40-13:00** **Hamal Hayatem**, *Estimates for Modified-Kantorovich Bernstein type rational operators based on weighted convergence* (online)

**13:00-13:20** **Florin Sofonea**, Ioan Țincu, *On a transformation of a sequence of real numbers*

## Lecture

Chairman: Rosario Corso

**15:30-15:50** **Gabriel Prajitura**, *On Rhaly Operators*

**15:50-16:10** **George Popescu**, *Rhaly operators and sub Cesaro operators*

**16:10-16:30** **Dorian Popa**, *On Ulam stability of a linear difference equation*

**16:30-16:50** **Daniela Marian**, *Hyers-Ulam stability of some partial integro-differential equation of order three*

**16:50-17:10** **Coffee break**

## Special Session for PhD students

Chairman: Maryam Mohammadi

**17:10-17:25 Ioana-Alexandra Șomîtcă**, *The approximation of the value of an inflection point on Bézier curves*

**17:25-17:40 Daniel Ianosi**, *Hermite-Hadamard Type Inequalities for Fractional Integrals*

**17:40-17:55 Sergiu-Vlad Pașca**, *The eigenstructure of certain operators*

**17:55-18:10 Gabriela Motronea, Alin Pepenar, Florin Sofonea**, *Rates of convergence for iterates of positive linear operators*

**18:10-18:25 Georgian-Cristian Chivu**, *Introduction into Image Segmentation Techniques based on AI approach, exploring advanced Techniques and Architectures*

**18:25-18:40 Gabriela-Denisa Motronea**, *Approximation functions of Steklov operators*

**19:30-22:00 Welcome Cocktail (Imparatul Romanilor Hotel)**

**Friday, July 19**

**Plenary Lecture**

**Chairman: Margareta Heilmann**

**09:00-09:40** José Antonio Adell, Daniel Cárdenas-Morales, *Strong converse inequalities for Bernstein polynomials with explicit constants*

**09:40-10:20** Ulrich Abel, Ana-Maria Acu, Margareta Heilmann, Ioan Raşa, *Cauchy problems and positive linear operators*

**Lecture**

**Chairman: José Antonio Adell**

**10:20-10:40** Ulrich Abel, Ana Maria Acu, Margareta Heilmann, Ioan Raşa, *Some results for a general class of Szász-Mirakjan-Durrmeyer operators*

**10:40-11:00** Ulrich Abel, Dany Leviatan, Ioan Raşa, *Characterizing convexity of functions by linear approximation processes*

**11:00-11:30** Coffee break

**Chairman: Ulrich Abel**

**11:30-11:50** Francisco Javier Martínez Sánchez, *Some approximation results on linear and positive bivariate operators under generalized convergence*

**11:50-12:10** Ralf Rigger, *On a teaching example for non-linear control*

**12:10-12:30 Dana Simian**, *On an approach for kernel selection in support vector machines*

## Lecture

**Chairman: Dorian Popa**

**14:30-14:50 Miruna-Ștefana Sorea**, *Morsifications of univariate real singularities - a combinatorial study*

**14:50-15:10 Voichita Adriana Radu**, *Baskakov operators of integral type*

**15:10-15:30 Emilia-Loredana Pop**, *The intermediate point from Bonnet theorem*

**15:30-16:00 Coffee break**

## PYTHAGORAS

**PROGRAMME Erasmus + ACTION TYPE  
KA220-HED - Cooperation partnership in higher  
education**

**16:00-16:30 Israel Garcia**, *WP1-Mini PBL*

**16:30-17:00 Georgios Triantafylidis**, *WP2 - Gamification in university math*

**17:00-17:30 Nicolae Constantinescu, Augusta Rațiu, Florin Sofonea, Oana Țicleanu**, *WP3 - Pythagoras - an improvement way for knowledge transfer*

## Special Session for PhD students

**Chairman: Voichita Adriana Radu**

**17:30-17:45 Andra-Mihaela Seserman**, *Composition and decomposition of positive linear operators (online)*

**17:45-18:00** **Andreea-Laura Burlacu**, *Continuous spectrum for an eigenvalue problem governed by the  $(p, q)$ -Laplacian*  
(online)

**18:00-18:15** **Ancuta Emilia Steopoaie**, *Strongly convex squared norms* (online)

**18:15-18:30** **Ioan Vladimir Vintu**, *Anti-periodic evolution inclusions of the second-order with one parameter*  
(online)

**19:30-22:00** **Official Dinner (Imparatul Romanilor Hotel)**

## **Saturday, July 20**

**09:30-13:30** **Excursion Sighisoara** (Departure from Sala Thalia, Cetății str., no. 3-5)

**14:00-15:30** **Lunch (Domeniul Dracula Danes)**

# Abstracts ICATA 2024

**Ulrich Abel, Dany Leviatan, Ioan Raşa**

**Title:** CHARACTERIZING CONVEXITY OF FUNCTIONS BY LINEAR APPROXIMATION PROCESSES

**Abstract.** About 30 years ago Ioan Raşa stated the following inequality involving Bernstein basis polynomials

$$p_{n,\nu}(x) = \binom{n}{\nu} x^\nu (1-x)^{n-\nu}$$

and convex functions as an open problem. Prove or disprove:

Let  $n \in \mathbb{N}$ . If  $f \in C[0, 1]$  is a convex function, then, for all  $x, y \in [0, 1]$ ,

$$\sum_{i=0}^n \sum_{j=0}^n [p_{n,i}(x)p_{n,j}(x) + p_{n,i}(y)p_{n,j}(y) - 2p_{n,i}(x)p_{n,j}(y)] f\left(\frac{i+j}{2n}\right) \geq 0.$$

In 2017, J. Mrowiec, T. Rajba and S. Wasowicz [5] affirmed the conjecture in positive. Their proof makes heavy use of probability theory. A short elementary proof and the corresponding results for Mirakyan–Favard–Szász operators and Baskakov operators can be found in [1].

In 2020, Abel and Leviatan [2] found a generalization of the inequality to  $q$ -monotone functions on  $[0, 1]$ , for any positive integer  $q$ . In 2022, Abel, Leviatan and Raşa [3] proved a converse for functions in  $C[0, 1]$ . The combination of both results provides a characterization of  $q$ -monotone functions in  $C[0, 1]$ .

Finally, we study the preservation of  $q$ -monotonicity of various Durrmeyer-type operators in  $[0, 1]$  or  $[0, \infty)$  as the case may be. These results yield characterizations of continuous  $q$ -monotone functions by the behavior of the integrals of the function with respect to measures that are related to the fundamental polynomials of the operators.

## References

- [1] Abel Ulrich, An inequality involving Bernstein polynomials and convex functions, *J. Approx. Theory* 222 (2017), 1–7.
- [2] Abel Ulrich, Dany Leviatan, An extension of Raşa’s conjecture to  $q$ -monotone functions, *Results Math.* 75:180 (2020), 1–13.
- [3] Abel Ulrich, Dany Leviatan, Ioan Raşa, Relations between the Bernstein polynomials and  $q$ -monotone functions, *Results Math.* 77:239 (2022), 1–12.
- [4] Abel Ulrich, Ioan Raşa, A sharpening of a problem on Bernstein polynomials and convex functions, *Math. Inequ. & Appl.* 21 (2018), 773–777.
- [5] Mrowiec Jacek, Teresa Rajba, Szymon Wasowicz, A solution to the problem of Raşa connected with Bernstein polynomials, *J. Math. Anal. Appl.* 446 (2017), 864–878.

## **José A. Adell, Daniel Cárdenas-Morales**

**Title:** STRONG CONVERSE INEQUALITIES FOR BERNSTEIN POLYNOMIALS WITH EXPLICIT CONSTANTS

**Abstract.** The Bernstein polynomials  $B_n f$  of a function  $f$  are the paradigmatic examples of positive linear operators. Since the 90’s, it is known that the rate of convergence from  $B_n f$  to  $f$  in the usual sup-norm is characterized as

$$K_1 \omega_2^\varphi \left( f, \frac{1}{\sqrt{n}} \right) \leq \|B_n f - f\| \leq K_2 \omega_2^\varphi \left( f, \frac{1}{\sqrt{n}} \right),$$

where  $\omega_2^\varphi(f, \cdot)$  stands for the Ditzian-Totik second modulus of smoothness with weight function  $\varphi(x) = \sqrt{x(1-x)}$ . Several estimates for the upper constant  $K_2$ , depending on the degree of smoothness of  $f$ , have been obtained by many authors. However, as far as we know, no explicit estimate for the lower constant  $K_1$  has been given, yet.

The first aim of this talk is to obtain upper and lower estimates for  $K_1$ . To do this, we use different tools, such as iterates of the operator  $B_n$ , together with their corresponding probabilistic representations, various expressions of the derivatives of  $B_n f$ , particularly



that in terms of the Krawtchouk polynomials (the orthogonal polynomials with respect to the binomial law), and accurate estimates of inverse moments of suitable random variables.

In the second place, we consider random Bernstein polynomials, which are obtained when we replace the equidistant deterministic nodes  $k/n, k = 0, 1, \dots, n$  by random nodes. We show that such random polynomials uniformly converge in probability to  $f$ , giving at the same time rates of convergence. These results are illustrated in the case that the random nodes are the uniform order statistics.

## Andreea-Laura Burlacu

**Title:** CONTINUOUS SPECTRUM FOR AN EIGENVALUE PROBLEM GOVERNED BY THE  $(p, q)$ -LAPLACIAN

**Abstract.** We study a two-phase eigenvalue problem governed by the  $(p, q)$ -Laplacian, with positive potentials and parametric boundary conditions. By employing the Nehari manifold method and variational techniques, we establish the existence of a nontrivial open interval  $I \subseteq \mathbb{R}$  such that every  $\lambda \in I$  is an eigenvalue of the problem under consideration.

### References

- [1] Barbu L., Burlacu A., Morosanu G., An eigenvalue problem involving the  $(p, q)$ -Laplacian with a parametric boundary condition, *Mediterr.J. Math.* (2023) 20:232.
- [2] Barbu L., Morosanu G., Full description of the eigenvalue set of the  $(p, q)$ -Laplacian with a Steklov-like boundary condition, *J. Differential Equations*, 290(2021), 1-16.
- [3] Barbu L., Burlacu A., Morosanu G., On an eigenvalue problem associated with the  $(p, q)$ -Laplacian, *An. St. Univ. Ovidius Constanta, Ser. Mat.*(2024).
- [4] Barbu L., Morosanu G., Eigenvalues of the negative  $(p, q)$ -Laplacian under a Steklov-like boundary condition, *Complex Var. Elliptic Equ.* 64(4), 685-700 (2019).

# Georgian-Cristian Chivu

**Title:** INTRODUCTION INTO IMAGE SEGMENTATION TECHNIQUES BASED ON AI APPROACH, EXPLORING ADVANCED TECHNIQUES AND ARCHITECTURES

**Abstract.** This study delves into the essential process of image segmentation, a key component of computer vision that involves dividing images into distinct segments to extract pertinent information. Image segmentation is critically important in fields such as medical image analysis, autonomous driving, and satellite imaging. The advent of deep learning, especially neural network models, has driven substantial advancements in image segmentation, significantly improving both accuracy and efficiency. Additionally, the foundational role of Convolutional Neural Networks (CNNs) is explored, with a focus on architectures such as U-Net, Mask R-CNN, and DeepLab, which have transformed the field by enabling precise localization and segmentation. Advanced techniques, such as dilated convolutions, attention mechanisms, and Conditional Random Fields (CRFs), are also being examined for their role in enhancing model performance by capturing multi-scale contextual information and refining segmentation outcomes. Furthermore, this study considers innovative approaches like Generative Adversarial Networks (GANs) for synthetic contrast enhancement of CT images, demonstrating the potential of deep learning in medical imaging to provide improved visualization and diagnostic capabilities. The study concludes by identifying future research directions, focusing on optimizing these techniques and exploring new architectures to address current challenges and enhance the generalizability and reliability of AI-driven image segmentation. We might also explore a software-based approach if we have access to a suitable dataset.

## References

[1] Ronneberger, O., Fischer, P., & Brox, T. (2015). U-Net: Con-

- volutional Networks for Biomedical Image Segmentation. In International Conference on Medical Image Computing and Computer-Assisted Intervention (MICCAI).
- [2] He, K., Gkioxari, G., Dollár, P., & Girshick, R. (2017). Mask R-CNN. In Proceedings of the IEEE International Conference on Computer Vision (ICCV).
- [3] Chen, L. C., Papandreou, G., Schroff, F., & Adam, H. (2018). Encoder-Decoder with Atrous Separable Convolution for Semantic Image Segmentation. In Proceedings of the European Conference on Computer Vision (ECCV).
- [4] Chen, L. C., Zhu, Y., Papandreou, G., Schroff, F., & Adam, H. (2018). DeepLab: Semantic Image Segmentation with Deep Convolutional Nets, Atrous Convolution, and Fully Connected CRFs. *IEEE Transactions on Pattern Analysis and Machine Intelligence*.
- [5] Zhao, H., Shi, J., Qi, X., Wang, X., & Jia, J. (2017). Pyramid Scene Parsing Network. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR).
- [6] Web source: Image Segmentation: Architectures, Losses, Datasets, and Frameworks <https://neptune.ai/blog/image-segmentation>
- [7] Web source: A fully automated pipeline for mining abdominal aortic aneurysm using image segmentation <https://www.nature.com/articles/s41598-019-50251-8>
- [8] Choi, J.W., Cho, Y.J., Ha, J.Y. et al. Generating synthetic contrast enhancement from non-contrast chest computed tomography using a generative adversarial network. *Sci Rep* 11, 20403 (2021). <https://doi.org/10.1038/s41598-021-00058-3>.
- [9] Liu J, Tian Y, Duzgol C, Akin O, Ağildere AM, Haberal KM, Coşkun M. Virtual contrast enhancement for CT scans of abdomen and pelvis. *Comput Med Imaging Graph*. 2022 Sep;100:102094. doi: 10.1016/j.compmedimag.2022.102094. Epub 2022 Jul 26. PMID: 35914340; PMCID: PMC10227907.
- [10] Web Dataset, <https://www.kaggle.com/datasets/hgunraj/covidxct>

## Rosario Corso

**Title:** MEAN SAMPLING KANTOROVICH OPERATORS

**Abstract.** As well-known, generalized sampling operators and sampling Kantorovich operators are able to approximate continuous signals and even  $L^p$ -signals in the latter case. Anyway, in the situation of a signal affected by a noise, these operators are not very efficient to approximate the cleaned signal (i.e., filtered by the noise) when the parameter goes to infinity. In order to solve this problem, we introduce a new type of operator, which we call the mean sampling Kantorovich operator, we study its approximation properties and made a comparison with the classical sampling Kantorovich operator in dealing with noised signals. This is a joint work with Gianluca Vinti.

## Valentin Gabriel Cristea

**Title:** NEW ESTIMATES FOR THE WALLIS' RATIO

**Abstract.** The aim of this work is to give an improvement of Mahmoud's in- equality from [M. Mahmoud, A. Talat, H. Moustafa, R.P. Agarwal, Completely monotonic functions involving Bateman's G-function, J. Comput. Anal. Appl., Vol. 29, No. 5, (2021), 970-986] and to show the lower and upper bounds for Wallis' ratio.

## Dorin Dumitrascu

**Title:** ON THE NON-HOLONOMIC CHARACTER OF FIRST TIME RETURNS IN THE EUCLIDEAN LATTICE

**Abstract.** We give precise asymptotics for the number of closed walks (excursions) in the standard Euclidean lattice. We also discuss the more complicated case of first time returns. We show that, in dimension greater or equal to 2, the sequence of excursions is P-recursive, transcendental over  $\mathbb{C}$ , but algebraic modulo primes, while the sequence of first time returns is not P-recursive. Techniques from the theory of holonomic functions, Fuchsian ODEs, and

analytic combinatorics are employed in the proofs. This presentation reports on joint work with Liviu Suciu.

## Hamal Hayatem

**Title:** ESTIMATES FOR MODIFIED-KANTOROVICH BERNSTEIN TYPE RATIONAL OPERATORS BASED ON WEIGHTED CONVERGENCE

**Abstract.** In this paper, we introduce a new modification of  $-q$  Balázs-Szabados operators and investigate the weighted approximation properties of a new modification of  $-q$  Bernstein-Kantorovich by using the weighted modulus of continuity and we provide the main convergence result for the weighted estimation of these new operators.

## Ulrich Abel, Ana Maria Acu, Margareta Heilmann, Ioan Raşa

**Title:** SOME RESULTS FOR A GENERAL CLASS OF SZÁSZ-MIRAKJAN-DURRMEYER OPERATORS

**Abstract.** We consider sequences of positive linear operators  $S_{n,j}$  which can be viewed as a generalization of the Szász-Mirakjan-Durrmeyer operators [1], Phillips operators [2] and corresponding Kantorovich modifications of higher order.

In our talk we present commutativity results for the operators, their commutativity with certain differential operators and some spectral properties.

### References

- [1] S. M. Mazhar, V. Totik, Approximation by modified Szász operators, Acta Sci. Math., 49, 257–269 (1985).
- [2] R. S. Phillips, An inversion formula for Laplace transforms and semi-groups of linear operators, Ann. Math. (2) 59 (1954), 325-356.

## Daniel Ianosı

**Title:** HERMITE-HADAMARD TYPE INEQUALITIES FOR FRACTIONAL INTEGRALS

**Abstract.** In this note we present necessary and sufficient conditions for Hermite-Hadamard type inequalities to occur for linear and positive functionals. In particular, the results from [ M. Z. Sarikaya, E. Set, H. Yaldiz, N. Bařak, Hermite–Hadamard’s inequalities for fractional integrals and related fractional inequalities, *Mathematical and Computer Modelling*, 57 (9–10), 2013, 2403–2407] are obtained.

## Daniela Marian

**Title:** HYERS-ULAM STABILITY OF SOME PARTIAL INTEGRO-DIFFERENTIAL EQUATION OF ORDER THREE

**Abstract.** We present some results regarding Hyers-Ulam and Hyers-Ulam-Rassias stability of some nonlinear partial integro-differential equation of order three. We give some illustrative examples to support the results. These types of equations appear in various applications in engineering, economics, chemistry, biology, etc.

## Maryam Mohammadi, Mohammad Heidari

**Title:** RBFs AS SURFACES OF REVOLUTION

**Abstract.** A common goal is to approximate a continuous function  $f : \mathbb{R}^d \rightarrow \mathbb{R}$ , by interpolating at centers  $X = \{x_j\}_{j=1}^n$ . This process employs finite-dimensional linear spaces of simpler functions. An effective and widely used technique involves interpolation through linear combinations of radial basis functions (RBFs)  $\varphi(r)$ , where  $r = \|x\|_2$  [1]. The interpolation expression is given by

$$s(x) = \sum_{j=1}^n \lambda_j \phi(\|x - x_j\|),$$

where the unknown coefficients are determined by satisfying the interpolation conditions

$$s(x_j) = f_j, \quad j = 1, \dots, n.$$

This results the linear system of equations  $A\lambda = f$ , where  $A = [\phi(\|x_i - x_j\|)]_{1 \leq i, j \leq n}$ . It is well-known that the RBF method is increasingly more accurate on steeper gradient surfaces and has difficulty approximating flat functions. The apparent reason is that for the scaled RBF  $\phi(\varepsilon r)$ , the flat surfaces are represented by linear combinations of vary small shape parameters  $\varepsilon$ . But as  $\varepsilon$  becomes small, so does the condition number. Flat surfaces are parts of planes, cones, or cylinders where the Gaussian curvatures are zero. So it seems that one can choose appropriate RBFs according to the geometric properties of the function to be approximated [2]. This requires that we parameterize RBFs by a regular parameterization like polar coordinates that leads to surfaces of revolution which are known for their simplicity in mathematical calculations and analysis. This can make it easier to study the surface's geometry, properties, and behavior.

In this talk, we first go through differential geometry basics [3]. Then we introduce the fundamental theorem of surface theory which describes the conditions for congruency of two parametrized surfaces. Then RBFs are categorized as surfaces of revolution according to the relation between their Gaussian and mean curvatures with the shape parameter  $\varepsilon$ . Some discussions are also given on the shape parameter selection of RBFs.

## References

- [1] H. Wendland. Scattered data approximation, volume 17. Cambridge university press, 2004.
- [2] M. Heidari, M. Mohammadi, S. De Marchi. Curvature based characterization of radial basis functions: application to interpolation. *Mathematical Modelling and Analysis*, **28**(3) (2023), 415-433.
- [3] B. O'neill. Elementary differential geometry. Elsevier, 2006.

## Francisco Javier Martínez Sánchez

**Title:** SOME APPROXIMATION RESULTS ON LINEAR AND POSITIVE BIVARIATE OPERATORS UNDER GENERALIZED CONVERGENCE

**Abstract.** In this talk, we present some classic Korovkin-type results about the convergence of double sequences of functions defined from sequences of positive linear operators in terms of generalized convergence. Specifically, we prove a qualitative Korovkin theorem and various quantitative versions, involved with various notions of bivariate modulus of continuity. Finally, as an application of the previous results, we show how an usual convergence in the literature is a particular case of our general setting.

## Gabriela-Denisa Motronea

**Title:** APPROXIMATION FUNCTIONS OF STEKLOV OPERATORS

**Abstract.** In this paper, we present a family of such operators depending on a parameter  $b > 0$ , Steklov type operators. The corresponding sequence  $(L_{n,b})_{n \geq 0}$  can be defined by a recurrence relation. It can be represented in terms of divided differences. Each operator  $L_{n,b}$  is a convolution-type operator and has an integral representation involving a B-spline function.

### References

- [1] Abel, U., Ivan, M.: Asymptotic approximation with a sequence of positive linear operators, *J Comput. Anal. Appl.* 3(4), 331-341 (2001).
- [2] M.Campiti, I. Rasa, C.Tacelli, Steklov operators and their associated semigroups, *Acta Sci. Math. (Szeged)*, 74- (2008), 171-189.
- [3] N.Merentes, K. Nikodem, remarks on strongly convex functions. *Aequ. Math.* 80, 193-199 (2010).
- [4] Popa, D., Rasa, I., Steklov Averages as Positive Linear Operators, *Filo-mat* 01/2016; 30(5):1195-1201. DOI:10.2298/FIL1605195P.



[5] O. Agratini, Aproximare prin operatori liniari, Presa Universitara Clujeana, Cluj-Napoca, 2000.

## Sergiu-Vlad Paşca

**Title:** THE EIGENSTRUCTURE OF CERTAIN OPERATORS

**Abstract.** The eigenstructure of certain positive linear operators is presented. The construction follows the technique used in the eigenstructure of classical Bernstein operators. Moreover, the limit of the recurrence relation for computing the coefficients among the eigenfunction is described.

## Gabriela Motronea, Alin Pepenar, Florin Sofonea

**Title:** RATES OF CONVERGENCE FOR ITERATES OF POSITIVE LINEAR OPERATORS

**Abstract.** We are concerned with positive linear operators defined on  $C(X)$ , where  $X$  is a simplex or a hypercube. We assume that the operators preserve the affine functions. After identifying an eigenvalue  $a \in [0, 1)$  of such an operator  $L$ , we show that the sequence  $(L^k f)_{k \geq 1}$  has a limit  $Vf$ ,  $f \in C(X)$  and  $|L^k f(x) - Vf(x)|$  is dominated by  $a^k$  multiplied by a factor depending on  $L$ ,  $f$  and  $x$ .

## Emilia-Loredana Pop

**Title:** THE INTERMEDIATE POINT FROM BONNET THEOREM

**Abstract.** Having as a start point the Bonnet theorem, two functions  $f$  and  $g$  continuous and monotone on the interval  $[a, b]$ , we consider the intermediate point functions  $c$  and  $\theta$  to be derivable in the point  $a$ . In what follows we establish conditions for the functions  $f$  and  $g$  such that the function  $\theta : (a, b) \rightarrow [0, 1]$  has limit

in the point  $x = a$  and the intermediate point function  $\bar{c}$  to be derivable in the point  $a$  and we provide its derivative  $\bar{c}'(a)$ .

## References

- [1] Dorel I. Duca, *Analiza Matematica* (vol. I), Casa Cartii de Stiinta, Cluj-Napoca, 2013.
- [2] Dorel I. Duca, Properties of the intermediate point from the Taylor's theorem, *Mathematical Inequalities & Applications*, vol. 12, no. 4, 2009, 423-432.
- [3] Dorel I. Duca, Ovidiu Pop, Concerning the Intermediate Point in the Mean- Value Theorem, *Mathematical Inequalities & Applications*, vol. 12, no. 3, 2009, 375-389.
- [4] Tiberiu Trif, Asymptotic Behavior of Intermediate Points in certain Mean Value Theorems, *Journal of Mathematical Inequalities*, vol. 2, no. 2, 2008, 151-161.

## Delia-Maria Kerekes, Dorian Popa

**Title:** ON ULAM STABILITY OF A LINEAR DIFFERENCE EQUATION

**Abstract.** An iterative method generates a sequence associated with an equation, that, under appropriate conditions, converges to a solution of that equation. A perturbation of the equation produces also a perturbation of the sequence. In this paper, we study the Ulam stability of an operatorial equation of the form  $x_{n+1} = T_n x_n + a_n$ , where  $T_n : X \rightarrow X$ ,  $n \in \mathbb{N}$ , are linear and bounded operators acting on a Banach space  $X$ . As applications we obtain some stability results for the sequences associated to some classical discrete and integral operators.

## References

- [1] Baias, A.R., Popa, D., Raşa, I.: Ulam stability of a successive approximation equation. *J. Fixed Point Theory Appl.*, **22** (2), 41, (2020).

- [2] Popa, D.: Hyers-Ulam-Rassias stability of the linear recurrence. *J. Math. Anal. Appl.*, **309**, 591–597, (2005).
- [3] Popa, D., Kerekeş, D.-M.: On Ulam Stability of an Operatorial Equation. *Mediterranean Journal of Mathematics* **18** (3), 118, (2021).
- [4] Xu, M., Hyers-Ulam-Rassias stability of a system of first order linear recurrences. *Bull. Korean Math. Soc.* **44** (4), 841–849, (2007).

## George Popescu

**Title:** RHALY OPERATORS AND SUB CESARO OPERATORS

**Abstract.** Rhaly operators on separable Hilbert spaces are defined by inferior triangular terraced matrices, generated by a sequence of complex numbers  $(a_n)_{n \geq 1}$ . Cesaro operator is the Rhaly operator generated by  $(1/n)_{n \geq 1}$ . Consider a subsequence  $(n_k)_{k \geq 1}$  of natural numbers  $(n)_{n \geq 1}$ , a Sub Cesaro operator is the Rhaly operator generated by the sequence  $(a_n)_{n \geq 1}$  where  $a_{n_k} = 1/n_k$  and all other  $a_n = 0$ .

We show there is strong connection between boundedness and compactness of a Rhaly operator and properties of Cesaro and Sub Cesaro operators, and hence coming down to properties of subseries  $\sum 1/n_k$  of the harmonic series  $\sum 1/n$ , which are related to open problems of such subseries, actually hard to solve.

## Gabriel Prajitura

**Title:** ON RHALY OPERATORS

**Abstract.** Rhaly operators are an abstractization and one possible generalization of the Cesaro operator. We will discuss some fundamental questions about these operators: boundedness, compactness, normality, spectrum. We will present some complete characterizations of these properties and some partial answers.

## Voichita Adriana Radu

**Title:** BASKAKOV OPERATORS OF INTEGRAL TYPE

**Abstract.** We propose a new integral Baskakov operators which reproduce a positive function that have  $f(0)=0$  and tent to infinity when  $x$  is large enough. For the constructed operator, some approximation properties are studied.

## Ulrich Abel, Ana-Maria Acu, Margareta Heilmann, Ioan Rasa

**Title:** CAUCHY PROBLEMS AND POSITIVE LINEAR OPERATORS

**Abstract.** A family of positive linear operators  $(S_{n,j})$  indexed by a real  $n > 0$  and depending on a parameter  $j$  is presented in the talk delivered by Margareta Heilmann at this Conference. Defining  $T_{t,j} := S_{1/t,j}$ ,  $t > 0$ , we get a family of operators with properties reminiscent of those of a  $C_0$ -semigroup. If it were a  $C_0$ -semigroup, it would solve a Cauchy problem. But it is not a  $C_0$ -semigroup. However, we study the associated Cauchy problem. The conclusions are presented in this talk.

## Ralf Rigger

**Title:** ON A TEACHING EXAMPLE FOR NONLINEAR CONTROL

**Abstract.** This paper evolved from a student teaching project in the non-linear control class: In the last 50 years numerous methods have been developed for dealing with the control of nonlinear dynamic systems. Starting for example with the approximation of such systems by linearisation and then using linear solution methods, continuing via the differential geometric approach of feedback linearisation, coming up to modern adaptive and model adaptive methods. Especially advanced adaptive control methods are becoming increasingly popular in various engineering applications.

Electro motors of any kind and the respective electro-mechanical power converters are examples of the successful application of this theory. A. M. Lyapunov introduced in his work on the stability of nonlinear differential equations a generalised notion of energy in 1892. From these basic ideas the area of non-linear controller and controlled system convergence has been developed. In the 1980's adaptive control for non-autonomous system advanced and was applied in real world applications successfully. In this paper we present a real world implementation of a simple adaptive current control for a R-L-Circuit with all parameters unknown. The purpose is to control the circuit's current to follow a prescribed signal and in order to do so, to estimate the resistance and inductance of the circuit. One particulate challenge was to construct a simple embedded controller and a lightweight R-L-Circuit, so that it can also be demonstrated in student classes. Since inductances only show up with increasing frequencies, we used a multi core controller, where the control algorithm runs on one core and the communication management is performed on another core. In this manner higher frequency signals can be handled easily and cleanly. In contrast to numerical simulations it will become instantly clear, that for such a control constraint of the R-L-Circuit come in to play and need to be considered in the demanded current signal.

## **Andra-Mihaela Seserman**

**Title:** COMPOSITION AND DECOMPOSITION OF POSITIVE LINEAR OPERATORS

**Abstract.** The idea behind Poisson approximation to the binomial distribution was used in [J. de la Cal, F. Luquin, J. Approx. Theory, 68(3), 1992, 322-329] and subsequent papers in order to establish the convergence of suitable sequences of positive linear operators. The proofs in these papers are given using probabilistic methods. We use similar methods, but in analytic terms. In this way we recover some known results and establish several new ones. In particular, we enlarge the list of the limit operators.

## Dana Simian

**Title:** ON AN APPROACH FOR KERNEL SELECTION IN SUPPORT VECTOR MACHINES

**Abstract.** This article proposes a method for optimizing kernel selection for Support Vector Machines (SVMs). SVMs are machine learning methods used for classification and regression, known for their good generalization power across many types of input data. However, one main drawback of SVMs is the large number of hyperparameters that must be chosen during the model selection step. The kernel function is one of the most important parameters and comes with a set of other internal parameters that must be optimized. We propose a method for building and optimizing complex kernels for classification and regression tasks based on evolutionary algorithms. We validated our approach on many datasets and compared the prediction results of the optimized SVM with other relevant machine learning methods. The reported results demonstrated the efficiency of our method.

## Florin Sofonea, Ioan Țincu

**Title:** ON A TRANSFORMATION OF A SEQUENCE OF REAL NUMBERS

**Abstract.** In this paper, we determine finite sections of a regular transformation, using random variables which follows the geometrical law, respectively Pascal's law and the central limit theorem.

## Miruna-Ștefana Sorea

**Title:** MORSIFICATIONS OF UNIVARIATE REAL SINGULARITIES - A COMBINATORIAL STUDY

**Abstract.** We focus on a large class of morsifications of germs of real analytic functions in one variable. We give a combinatorial

characterization of the resulting Morse functions based on the interplay between the Newton-Puiseux roots of the polar curves of the morsifications and those of the discriminant curves. This is based on my joint work with Arnaud Bodin (University of Lille, France), Evelia Rosa García Barroso (University of La Laguna, Spain) and Patrick Popescu-Pampu (University of Lille, France).

## Ioana-Alexandra Șomîtcă

**Title:** THE APPROXIMATION OF THE VALUE OF AN INFLECTION POINT ON BÉZIER CURVES

**Abstract.** The purpose of my presentation is to analyze the value of the inflection point on a Bézier curve of degree 3 and a Bézier curve of degree 4, using the derivative of order 2.

I will also use triangle areas in order to find the smallest interval in which the parameter "t" in the parametric formula of the Bézier curve lies.

The goal is to find a general algorithm for approximately locating the inflection point on the Bézier Curve of degree greater than 3.

## Ancuta Emilia Steopoaie

**Title:** STRONGLY CONVEX SQUARED NORMS

**Abstract.** Normed spaces for which the squared norm is strongly convex are intensively studied in literature. This note is motivated by the fact that a strongly convex squared norm plays a role in quantitative Korovkin approximation. We are concerned especially with the strong convexity of  $\|\cdot\|_2^p$  on  $\mathbb{R}^2$ ,  $1 < p < 2$ .

## Arianna Travaglini, Gianluca Vinti

**Title:** NONLINEAR SAMPLING DURRMEYER OPERATOR AND APPLICATIONS

**Abstract.** The talk is focused on some recent approximation results for the nonlinear version of Durrmeyer-sampling type operators and applications in the biomedical field. For what concerns the space of continuous functions, a pointwise and uniform convergence theorem for the nonlinear Durrmeyer-sampling type operator is provided. Moreover, approximation results in the general setting of Orlicz spaces are also discussed. The above mentioned family of operators, introduced in [1] in their linear version and studied, e.g., in [4], represents an extension of other well-known families of sampling operators, including both generalized and Kantorovich ones (see, e.g., [2,3]). Convergence and approximation results in multidimensional settings for the sampling Kantorovich operators, in their linear version, represent the theory at the basis of applications in the field of digital image reconstruction and processing ([6]). These operators, acting both as rescaling algorithms and low-pass filters, enhance reconstructed images by increasing information content and reducing noise disturbances. For what concerns applications in the biomedical field, three different studies will be considered: a first one concerns the identification of Alzheimer's disease biomarkers in MRI images ([5]), a second study is related to the extraction of the aorta artery patent lumen's from CT images of patients with atheromatous disease ([7]), and a third one consists in segmenting eye fundus images for cluster counting analysis in healthy and diabetic individuals ([8]). In all three of these applications, the fundamental role played by the reconstruction with the sampling Kantorovich operator will be highlighted.

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## Gianluca Vinti

**Title:** DISCRETE AND SEMI-DISCRETE SAMPLING TYPE OPERATORS AND THEIR APPLICATIONS TO IMAGE SEGMENTATION PROBLEMS

**Abstract.** In this talk I will present some approximation results for discrete and semi-discrete operators of sampling type  $([1, 2])$  and I will present their applications to digital image processing. Namely, I will consider a family of these operators to solve an image segmentation problem in the biomedical field using two approaches: a deterministic one ([3]), that exploits results of approximation theory, and a machine learning one, which benefits from the use of deep neural networks. Finally, the comparison of the two approaches will be discussed.

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## Ioan Vladimir Vîntu

**Title:** ANTI-PERIODIC EVOLUTION INCLUSIONS OF THE SECOND-ORDER WITH ONE PARAMETER

**Abstract.** In a real Hilbert space, we consider the reduced problem

$$(P_0)_{ap} \begin{cases} u'(t) + Au(t) + Bu(t) \ni f(t) \text{ for a.e. } t \in (0, T), \\ u(0) + u(T) = 0. \end{cases}$$

as well as its Lions regularization,

$$(P_\varepsilon)_{ap} \begin{cases} -\varepsilon u''(t) + u'(t) + Au(t) + Bu(t) \ni f(t) \text{ for a.e. } t \in (0, T), \\ u(0) + u(T) = 0, \quad u'(0) + u'(T) = 0, \end{cases}$$

where  $T > 0$  is a given time instant, the function  $f \in L^2(0, T; H)$  and the operators  $A = \partial\varphi$ ,  $B = -\partial\psi$  are the subdifferentials of some proper, convex and lower semicontinuous functions  $\varphi, \psi : H \rightarrow (-\infty, +\infty]$ .

Our aim is the investigation of strong solutions to problems  $(P_\varepsilon)_{ap}$  and  $(P_0)_{ap}$ , as well as the behavior of the solutions to problem  $(P_\varepsilon)_{ap}$  with respect to  $\varepsilon$ . Here, we also provide sufficient conditions for the uniqueness of these solutions. We also derive the continuous dependence of the solution to problem  $(P_\varepsilon)_{ap}$  on parameter  $\varepsilon$ , along

with approximation results for the solution to  $(P_0)_{ap}$ . The final part is reserved for applications in the forms of semilinear heat equations and systems of ordinary differential equations.

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## Participations to ICATA 2024

No. Crt.	Name/ e-mail	Affiliation
1	<b>Abel Ulrich</b> ulrich.abel@mnd.thm.de	Technische Hochschule Mittelhessen Germany
2	<b>Acu Ana Maria</b> anamaria.acu@ulbsibiu.ro	Lucian Blaga University of Sibiu, Romania
3	<b>Adell José Antonio</b> adell@unizar.es	University of Zaragoza, Spain
4	<b>Barza Sorina</b> barzasorina1@gmail.com	Karlstad University, Sweden
5	<b>Burlacu (Iordachianu) Andreea-Laura</b> andreea.laura.burlacu@gmail.com	Ovidius University of Constanta Romania
6	<b>Cardenas-Morales Daniel</b> cardenas@ujaen.es	University of Jaen Spain
7	<b>Chivu Georgian-Cristian</b> georgian.chivu@ulbsibiu.ro	Lucian Blaga University of Sibiu Romania
8	<b>Constantinescu Nicolae</b> nicolae.constantinescu@ulbsibiu.ro	Lucian Blaga University of Sibiu Romania
9	<b>Corso Rosario</b> rosario.corso02@unipa.it	University of Palermo Italy
10	<b>Cristea Valentin Gabriel</b> valigabi.cristea@gmail.com	Branesti Secondary School Romania
11	<b>Dumitrascu Dorin</b> ddumitrascu@adrian.edu	Adrian College United States
12	<b>Garcia Israel</b> igarcial@ull.edu.es	Universidad de La Laguna Spain
13	<b>Girjoaba Adrian</b> adrian.girjoaba@ulbsibiu.ro	Lucian Blaga University of Sibiu Romania
14	<b>Hayatem Hamal</b> hayathmal84@gmail.com	Tripoli University Libya
15	<b>Heidari Mohammad</b> Std_M.Heidari@khu.ac.ir	Kharazmi University Iran
16	<b>Heilmann Margareta</b> heilmann@math.uni-wuppertal.de	University of Wuppertal Germany
17	<b>Ianos Daniel</b> daniel.ianos@gmail.com	Technical University of Cluj-Napoca Romania
18	<b>Kerekes Delia-Maria</b> Delia.Kerekes@math.utcluj.ro	Technical University of Cluj-Napoca Romania
19	<b>Marian Daniela</b> daniela.marian@math.utcluj.ro	Technical University of Cluj-Napoca Romania
20	<b>Martínez Sánchez Francisco Javier</b> fjms0018@red.ujaen.es	University of Jaén Spain
21	<b>Mohammadi Maryam</b> m.mohammadi@khu.ac.ir maryam.mohammadi@unipd.it	Kharazmi University, Iran University of Padova, Italy
22	<b>Motronea Gabriela</b> gdenisa19@gmail.com	Technical University of Cluj-Napoca Romania
23	<b>Paşca Sergiu-Vlad</b> sergiu.pasca@ulbsibiu.ro	Lucian Blaga University of Sibiu Romania

## Participations to ICATA 2024

No. Crt.	Name/ e-mail	Affiliation
24	<b>Pepenar Alin</b> alin.pepenar@ulbsibiu.ro	Lucian Blaga University of Sibiu Romania
25	<b>Pop Emilia-Loredana</b> lorypel@gmail.com	Babes-Bolyai University Romania
26	<b>Popa Dorian</b> popa.dorian@math.utcluj.ro	Technical University of Cluj-Napoca Romania
27	<b>Popescu George</b> grgpop@gmail.com	University of Craiova, Romania
28	<b>Prajitura Gabriel</b> gabrielprajitura@gmail.com	State University of New York United States
29	<b>Radu Voichita Adriana</b> voichita.radu@econ.ubbcluj.ro	Babes-Bolyai University Romania
30	<b>Rasa Ioan</b> ioan.rasa@math.utcluj.ro	Technical University of Cluj-Napoca Romania
31	<b>Ratiu Augusta</b> augusta.ratiu@ulbsibiu.ro	Lucian Blaga University of Sibiu Romania
32	<b>Rigger Ralf</b> ralf.rigger@mnd.thm.de	Technische Hochschule Mittelhessen Germany
33	<b>Seserman Andra-Mihaela</b> campan_aandra@yahoo.com	Technical University of Cluj-Napoca Romania
34	<b>Simian Dana</b> dana.simian@ulbsibiu.ro	Lucian Blaga University of Sibiu Romania
35	<b>Sofonea Florin</b> florin.sofonea@ulbsibiu.ro	Lucian Blaga University of Sibiu Romania
36	<b>Sorea Miruna-Ştefana</b> mirunastefana.sorea@ulbsibiu.ro	Lucian Blaga University of Sibiu Romania
37	<b>Şomîtcă Ioana-Alexandra</b> ioana.somitca@gmail.com	Technical University of Cluj-Napoca Ştefan cel Mare University of Suceava Romania
38	<b>Steopoaie Ancuta Emilia</b> ancuta.steopoaie@gmail.com	Technical University of Cluj-Napoca Romania
39	<b>Travaglini Arianna</b> arianna.travaglini@unifi.it	University of Florence University of Perugia University of Basilicata Italy
40	<b>Triantafylidis Georgios</b> gt@create.aau.dk	Aalborg Universitet Denmark
41	<b>Ticleanu Oana</b> oana.ticleanu@ulbsibiu.ro	Lucian Blaga University of Sibiu Romania
42	<b>Tincu Ioan</b> ioan.tincu@ulbsibiu.ro	Lucian Blaga University of Sibiu Romania
43	<b>Vinti Gianluca</b> gianluca.vinti@unipg.it	University of Perugia Italy
44	<b>Vintu Ioan Vladimir</b> vladimir.vintu98@gmail.com	Ovidius University Constanta Romania