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Splitting the structured paths in stratified graphs

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Abstract

The concept of stratified graph introduce some method of knowledge representation ([7], [4]). The inference process developed for this method uses the paths of the stratified graphs, an order between the elementary arcs of a path and some results of universal algebras. The order is defined by considering a structured path instead of a regular path. In this paper we give two splitting properties. First property shows that every structured path can be uniquely decomposed by means of two structured subpaths. A similar decomposition is shown for the accepted structured paths. The decomposition of the accepted structured paths is used to define the inference process allowed by a stratified graph. This process is restated in the vision of the new results presented in this paper. This description is included in a separate section, where we define the concept of knowledge processing system based on stratified graphs. We give a formalism for the inference process in such systems.
Keywords: Peano algebra, labeled graph, stratified graph, structured path, accepted structured path, inference process

1 Introduction

The concept of stratified graph provides a method of knowledge representation. This concept was introduced in paper [7]. The resulting method uses concepts from graph theory redefined in the new framework and elements of universal algebra. Intuitively, a stratified graph is built over a labeled graph G_0, placing on top a subset of a Peano algebra generated by the label set of G_0.

The concept of structured path over a labeled graph was introduced in [4]. In the same paper was introduced the concept of accepted structured path over a stratified graph. The inference process was defined by means of a decomposition property of the accepted structured path, described in an intuitive manner in [4].

In this paper we define in a mathematical manner the concept of decomposition and obtain two splitting properties: one for a stratified graph and the other for an accepted structured path.

The inference process developed by a stratified graph is based on the decomposition of an accepted structured path into two accepted structured paths. These two components are subpaths of the initial path and the decomposition is iterated until we obtain atomic accepted paths. A subpath of a path defines a continuous path which consists of different kinds of elementary arcs of the initial path. Further we use the order induced by the structure of the accepted path and some meaning attached to every elementary arc.

The paper is organized as follows: Section 2 contains basic concepts as labeled graph and stratified graph. In section 3 we define the concept of structured path in a labeled graph, the concept of accepted structured path in a structured graph and we establish an useful result concerning the existence of some morphism of universal algebras obtained from the labels of the structured paths to Peano algebra generated by the elementary labels the structured graph (Proposition 4); Section 4 treats two decompositions of structured paths and accepted structured paths respectively; Section 5 defines the concepts of knowledge processing system based on stratified graphs and we give the formalism of the coresponding inference process. Last section includes conclusions of our study.

2 Basic concepts

We begin this section by a short presentation of two concepts: labeled graph and stratified graph. Various papers ([4], [3], [6]) present in their introduction these concepts. By *a labeled graph* we understand a tuple $G = (S, L_0, T_0, f_0)$, where S is a finite set of nodes, L_0 is a set of elements named *labels*, T_0 is a set of binary relations on S and $f_0 : L_0 \longrightarrow T_0$ is a surjective function. Such a structure admits a graphical representation. Each element of S is represented by a rectangle specifying the corresponding node. We draw an arc from $x_1 \in S$ to $x_2 \in S$ and this arc is labeled by $a \in L_0$ if $(x_1, x_2) \in f_0(a)$. This case is shown in Figure 1.

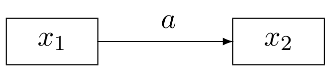

Figure 1: A labeled arc

We consider a symbol σ of arity 2 and take the sets defined recursively as follows:

$$\begin{cases} B_0 = L_0 \\ B_{n+1} = B_n \cup \{\sigma(x_1, x_2) \mid (x_1, x_2) \in B_n \times B_n\}, n \geq 0 \end{cases}$$

where L_0 is a finite set that does not contain the symbol σ. The set $\mathcal{B} = \bigcup_{n \geq 0} B_n$ is the Peano σ -algebra ([1]) generated by L_0. We can understand that $\sigma(x, y)$ is the word σxy over the alphabet $L_0 \cup \{\sigma\}$. Often this algebra is denoted by $\overline{L_0}$.

By $Initial(\overline{L_0})$ we denote a collection of subsets of B satisfying the following conditions: $M \in Initial(\overline{L_0})$ if

- $L_0 \subseteq M \subseteq B$
- if $\sigma(u, v) \in M$, $u \in \overline{L_0}$, $v \in \overline{L_0}$ then $u \in M$ and $v \in M$

We define the mapping $prod_S : dom(prod_S) \longrightarrow 2^{S \times S}$ as follows:

$$dom(prod_S) = \{(\rho_1, \rho_2) \in 2^{S \times S} \times 2^{S \times S} \mid \rho_1 \circ \rho_2 \neq \emptyset\}$$

$$prod_S(\rho_1, \rho_2) = \rho_1 \circ \rho_2$$

where $\circ$ is the usual operation between the binary relations:

$$\rho_1 \circ \rho_2 = \{(x,y) \in S \times S \mid \exists z \in S : (x,z) \in \rho_1, (z,y) \in \rho_2\}$$

We denote by $R(prod_S)$ the set of all the restrictions of the mapping $prod_S$:

$$R(prod_S) = \{u \mid u \prec prod_S\}$$

where $u \prec prod_S$ means that $dom(u) \subseteq prod_S$ and $u(\rho_1, \rho_2) = prod_S(\rho_1, \rho_2)$ for $(\rho_1, \rho_2) \in dom(u)$.

If u is an element of $R(prod_S)$ then we denote by $Cl_u(T_0)$ the *closure* of T_0 in the partial algebra $(2^{S\times S}, \{u\})$. This is the smallest subset Q of $2^{S\times S}$ such that $T_0 \subseteq Q$ and Q is closed under u. It is known that this is the union $\bigcup_{n\geq 0} X_n$, where

$$\begin{cases} X_0 = T_0 \\ X_{n+1} = X_n \cup \{u(\rho_1, \rho_2) \mid (\rho_1, \rho_2) \in dom(u) \cap (X_n \times X_n)\}, n \geq 0 \end{cases}$$

If $L \in Initial(L_0)$ then the pair $(L, \{\sigma_L\})$, where

- $dom(\sigma_L) = \{(x,y) \in L \times L \mid \sigma(x,y) \in L\}$
- $\sigma_L(x,y) = \sigma(x,y)$ for every $(x,y) \in dom(\sigma_L)$

is a partial algebra. This property is used to define the concept of stratified graph.

Consider a labeled graph $G_0 = (S, L_0, T_0, f_0)$. A *stratified graph* ([7]) $\mathcal{G}$ over G_0 is a tuple (G_0, L, T, u, f) where

- $L \in Initial(\overline{L_0})$
- $u \in R(prod_S)$ and $T = Cl_u(T_0)$
- $f : (L, \{\sigma_L\}) \longrightarrow (2^{S\times S}, \{u\})$ is a morphism of partial algebras such that $f_0 \prec f$, $f(L) = T$ and if $(f(x), f(y)) \in dom(u)$ then $(x,y) \in dom(\sigma_L)$

The existence of this structure, as well as the uniqueness is proved in [7]:

Proposition 1 *For every labeled graph $G_0 = (S < L_), T_0, f_0)$ and every $u \in R(prod_S)$ there is just one stratified graph (G_0, L, T, u, f) over G_0.*

3 Accepted structured paths

We consider a labeled graph $G_0 = (S, L_0, T_0, f_0)$. A *regular path* over G_0 is a pair $([x_1, \ldots, x_{n+1}], [a_1, \ldots, a_n])$ such that $(x_i, x_{i+1}) \in f_0(a_i)$ for every $i \in \{1, \ldots, n\}$.

Definition 1 *We denote by $STR(G_0)$ the smallest set satisfying the following conditions:*

- *For every $a \in L_0$ and $(x,y) \in f_0(a)$ we have $([x,y], a) \in STR(G_0)$.*
- *If $([x_1, \ldots, x_k], u) \in STR(G_0)$ and $([x_k, \ldots, x_n], v) \in STR(G_0)$ then $([x_1, \ldots, x_k, \ldots, x_n], [u,v]) \in STR(G_0)$.*

The concept of structured path introduces some order between the arcs taken into consideration for an regular path. To highlight the role of structured paths we consider the following example presented in Figure 2. We relieved here two structured paths: one of them is denoted by (1) and represents the structured path $([x_1, x_2, x_3, x_4], [[a_1, b_1], c_1])$; the other is denoted by (2) and represents the structured path $([x_1, x_2, x_3, x_4], [a_1, [b_1, c_1]])$. In order to explain in an intuitive manner the inference process we assign an algorithm to every arc symbol. For example, consider the following simple case: each arc symbol designates the following algorithm:

Alg

Input: x, y

Output: If $x \geq y$ then $x + y$; otherwise $x - y$

end

Each node of the labeled graph represents a natural number. In order to make a choice we take $x_1 = 7$, $x_2 = 2$, $x_3 = 5$ and $x_4 = 4$. For example the output of the algorithm for x_1 and x_2 is 9. We write $Alg(x_1, x_2) = 9$. For the paths (1) and (2) from Figure 2 we obtain

$$Alg(Alg(x_1, x_2), Alg(x_2, x_3)) = 14;\ Alg(Alg(Alg(x_1, x_2), Alg(x_2, x_3)), x_4) = 18$$

$$Alg(x_2, x_3) = -3;\ Alg(x_3, x_4) = 9;\ Alg(Alg(x_2, x_3), Alg(x_3, x_4)) = -12;$$

$$Alg(x_1, Alg(Alg(x_2, x_3), Alg(x_3, x_4))) = Alg(7, -12) = -5$$

Thus the inference process gives 18 for the first path and -5 for the second path.

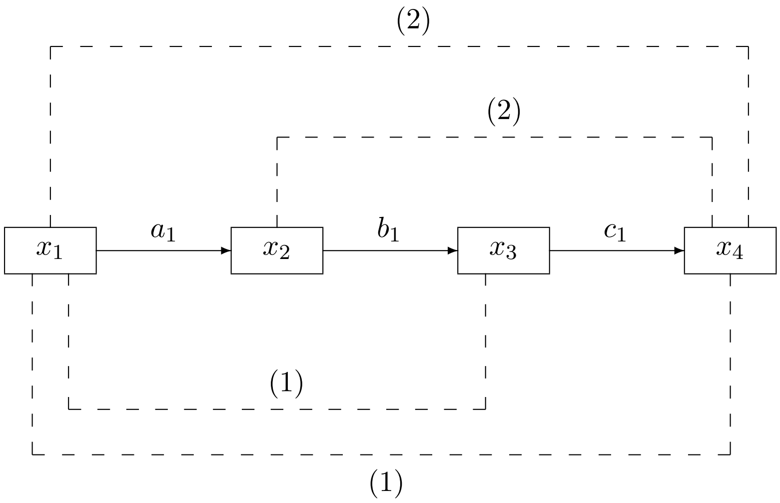

Figure 2: Intuitive representation of structured paths

Let us consider the set $\mathcal{L}(X) = \{[x_1, \ldots, x_n] \mid n \geq 1, x_i \in X, i = 1, \ldots, n\}$, the set of all nonempty lists over X. We denote $first([x_1, \ldots, x_n]) = x_1$ and $last([x_1, \ldots, x_n]) = x_n$.

We define the mapping

$$\otimes : STR(G_0) \times STR(G_0) \longrightarrow STR(G_0)$$

as follows:

- $dom(\otimes) = \{((\alpha_1, u_1), (\alpha_2, u_2)) \mid (\alpha_1, u_1) \in STR(G_0), (\alpha_2, u_2) \in STR(G_0),$
  $last(\alpha_1) = first(\alpha_2)\}$
- If $([x_1, \ldots, x_k], u) \in STR(G_0)$ and $([x_k, \ldots, x_n], v) \in STR(G_0)$ then

$$([x_1, \ldots, x_k], u) \otimes ([x_k, \ldots, x_n], v) = ([x_1, \ldots, x_n], [u, v])$$

Proposition 2 *Consider a labeled graph $G_0 = (S, L_0, T_0, f_0)$ and the set*

$$K(G_0) = \{([x, y], a) \mid (x, y) \in f_0(a)\} \tag{1}$$

The set $STR(G_0)$ is the $\otimes$-Peano algebra generated by $K(G_0)$.

Proof. From Definition 1 we deduce that $STR(G_0)$ is the smallest set containing $K(G_0)$ and closed under $\otimes$ operation. It follows that $STR(G_0)$ is the $\otimes$-Peano algebra generated by $K(G_0)$. ∎

We define

$$STR_2(G_0) = \{w \mid \exists(\alpha, w) \in STR(G_0)\}$$

In fact, $STR_2(G_0)$ represents the projection of the set $STR(G_0)$ on the second axis: in a classical notation we write $STR_2(G_0) = pr_2(STR(G_0))$.

We define the mapping $* : STR_2(G_0) \times STR_2(G_0) \longrightarrow STR_2(G_0)$ as follows:

- $dom(*) = \{(\beta_1, \beta_2) \mid \exists \alpha_1, \alpha_2 : (\alpha_1, \beta_1) \in STR(G_0), (\alpha_2, \beta_2) \in STR(G_0),$
  $last(\alpha_1) = first(\alpha_2)\}$
- If $\beta_1, \beta_2 \in dom(*)$ then $\beta_1 * \beta_2 = [\beta_1, \beta_2]$

Remark 1 *The pair $STR_2(G_0), *)$ becomes a partial algebra.*

Proposition 3 *$STR_2(G_0)$ is the $*$-Peano algebra generated by L_0.*

Proof. The set $STR(G_0)$ is the $\otimes$-Peano algebra generated by $K(G_0)$. This means that $STR(G_0) = \bigcup_{n \geq 0} M_n$, where

$$\begin{cases} M_0 = K(G_0) \\ M_{n+1} = M_n \cup \{\gamma \mid \exists(\alpha, \beta) \in dom(\otimes) \cap (M_n \times M_n), \gamma = \alpha \otimes \beta\} \end{cases} \tag{2}$$

It follows that

$$STR_2(G_0) = pr_2 STR(G_0) = pr_2(\bigcup_{n \geq 0} M_n) = \bigcup_{n \geq 0} pr_2 M_n =$$

$$pr_2 M_0 \cup \bigcup_{n \geq 0} pr_2 M_{n+1} = pr_2 K(G_0) \cup \bigcup_{n \geq 0} pr_2 M_{n+1} =$$

therefore

$$STR_2(G_0) = L_0 \cup \bigcup_{n \geq 0} pr_2 M_{n+1} \tag{3}$$

Based on (2) we obtain

$$pr_2 M_{n+1} = pr_2 M_n \cup pr_2 X_n \tag{4}$$

where $X_n = \{\gamma \mid \exists(\alpha,\beta) \in dom(\otimes) \cap (M_n \times M_n), \gamma = \alpha \otimes \beta\}$.
From (4) we find that

$$pr_2 M_{n+1} = pr_2 M_n \cup \{pr_2\gamma \mid \exists(\alpha,\beta) \in dom(\otimes) \cap (M_n \times M_n), \gamma = \alpha \otimes \beta\} \qquad (5)$$

Consider an element $\gamma \in X_n$. There are $(\alpha,\beta) \in dom(\otimes) \cap (M_n \times M_n)$ such that $\gamma = \alpha \otimes \beta$. This means that $\alpha = ([x_1,\ldots,x_k],u_1)$, $\beta = ([x_k,\ldots,x_m],v_1)$ and $\gamma = ([x_1,\ldots,x_k,\ldots,x_m],[u_1,v_1])$. It follows that $pr_2\gamma = [u_1,v_1]$ and by the definition of the operation $*$ we have $[u_1,v_1] = u_1 * v_1$. Thus, if $\gamma = \alpha \otimes \beta$, where $(\alpha,\beta) \in M_n \times M_n$ then $pr_2\gamma = pr_2\alpha * pr_2\beta$. This property allows to rewrite (6) as follows

$$pr_2 M_{n+1} = pr_2 M_n \cup \{w \mid \exists(\alpha,\beta) \in dom(\otimes) \cap (M_n \times M_n), w = pr_2\alpha * pr_2\beta\} \qquad (6)$$

Let us denote $Y_n = pr_2 M_n$ for every $n \geq 0$. We have $Y_0 = pr_2 M_0 = pr_2 K(G_0) = L_0$ and from (6) we obtain Let us prove that

$$\{w \mid \exists(\alpha,\beta) \in dom(\otimes) \cap (M_n \times M_n), w = pr_2\alpha * pr_2\beta\} = \{\omega \mid \exists(u,v) \in (Y_n \times Y_n) \cap dom(*) : \omega = u * v\} \qquad (7)$$

Take $w = pr_2\alpha * pr_2\beta$ for some $(\alpha,\beta) \in dom(\otimes) \cap (M_n \times M_n)$. It follows that $\alpha = ([x_1,\ldots,x_k], pr_2\alpha)$, $\beta = ([y_1,\ldots,y_r], pr_2\beta$ and $x_k = y_1$. Denote $pr_2\alpha = u$ and $pr_2\beta = v$. Because $\alpha = ([x_1,\ldots,x_k], pr_2\alpha) \in M_n$ we obtain $u = pr_2\alpha \in pr_2 M_n$. Similarly we have $v \in pr_2 M_n$. But $pr_2 M_n = Y_n$, therefore $u \in Y_n$ and $v \in Y_n$. We have $w = u * v$ therefore we proved the inclusion

$$\{w \mid \exists(\alpha,\beta) \in dom(\otimes) \cap (M_n \times M_n), w = pr_2\alpha * pr_2\beta\} \subseteq \{\omega \mid \exists(u,v) \in (Y_n \times Y_n) \cap dom(*) : \omega = u * v\} \qquad (8)$$

We prove now the converse inclusion. To prove this property we consider an element $\omega = u * v$ for some $(u,v) \in (Y_n \times Y_n) \cap dom(*)$. But $Y_n = pr_2 M_n$ and $u \in Y_n$. It follows that there is $\alpha = ([x_1,\ldots,x_k],u) \in M_n$ and $\beta \in ([y_1,\ldots,y_m],v) \in M_n$ $x_k = y_1$. We deduce that $(\alpha,\beta) dom(\otimes) \cap (M_n \times M_n)$ such that $\omega = pr_2\alpha * pr_2\beta$. This shows that

$$\{w \mid \exists(\alpha,\beta) \in dom(\otimes) \cap (M_n \times M_n), w = pr_2\alpha * pr_2\beta\} \supseteq \{\omega \mid \exists(u,v) \in (Y_n \times Y_n) \cap dom(*) : \omega = u * v\} \qquad (9)$$

Now, from (8) and (9) we obtain (7).
From (6) and (7) we obtain

$$pr_2 M_{n+1} = pr_2 M_n \cup \{\omega \mid \exists(u,v) \in (Y_n \times Y_n) \cap dom(*) : \omega = u * v\}$$

equivalently we can write that

$$Y_{n+1} = Y_n \cup \{\omega \mid \exists(u,v) \in (Y_n \times Y_n) \cap dom(*) : \omega = u * v\} \qquad (10)$$

From $Y_0 = L_0$ and (10) we obtain that $\bigcup_{n\geq 0} Y_n = \overline{L_0}$, where $\overline{L_0}$ is taken under operation $*$. From (3) we obtain $STR_2(G_0) = \bigcup_{n\geq 0} Y_n$, therefore $STR_2(G_0) = (\overline{L_0})_*$ and the proposition is proved. ■

Proposition 4 *The mapping* $h : (STR_2(G_0), *) \longrightarrow ((\overline{L_0})_\sigma, \sigma)$ *defined by*

$$h(p) = \begin{cases} p \text{ if } p \in L_0 \\ \sigma(h(u), h(v)) \text{ if } p = [u,v], u \in STR_2(G_0), v \in STR_2(G_0) \end{cases}$$

is a morphism of partial algebras. In other words, the diagram from Figure 3 is commutative.

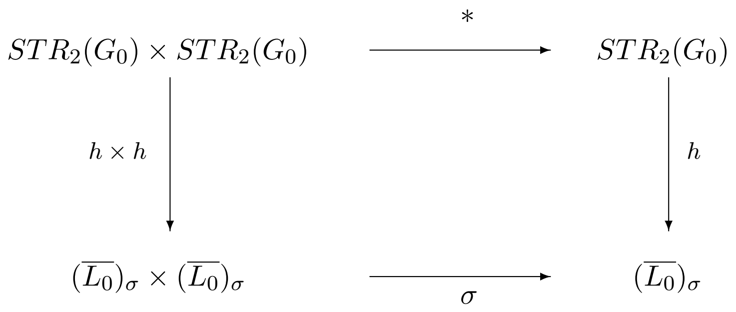

Figure 3: Commutative diagram

Proof. Consider $(u, v) \in dom(*)$. There are $([x_1, \ldots, x_k], u) \in STR(G_0)$ and $([x_k, \ldots, x_n], v) \in STR(G_0)$. If this is the case then $u * v = [u, v] \in STR_2(G_0)$ and $h([u, v]) = \sigma(h(u), h(v))$. Thus the diagram is commutative. ∎

Definition 2 *We define the set $ASP(\mathcal{G})$ as follows: $([x_1, \ldots, x_{n+1}], c) \in ASP(\mathcal{G})$ if and only if $([x_1, \ldots, x_{n+1}], c) \in STR(G_0)$ and $h(c) \in L$.*
An element of $ASP(\mathcal{G})$ is named ***accepted structured path*** *over $\mathcal{G}$.*

4 Splitting properties

In this section we obtain two splitting properties: one of them refers to the decomposition of a structured path; the other gives the decomposition of an accepted structured path. The first splitting property is used to prove the second property.

Proposition 5 *(splitting property I)*
If $([x_1, \ldots, x_{n+1}], c) \in STR(G_0)$ and $n \geq 2$ then there are $u, v \in STR_2(G_0)$ and $k \in \{2, \ldots, n\}$, uniquely determined, such that
$c = [u, v]$
$([x_1, \ldots, x_k], u) \in STR(G_0)$
$([x_k, \ldots, x_{n+1}], v) \in STR(G_0)$

Proof. We denote by $(\overline{L_0})_*$ the *-Peano algebra generated by L_0. By Proposition 3 we have $STR_2(G_0) = (\overline{L_0})_*$. In a similar manner we consider the $\otimes$-Peano algebra generated bu $K(G_0)$, denoted by $(\overline{K(G_0)})_\otimes$. By Proposition 2 we have $STR(G_0) = (\overline{K(G_0)})_\otimes$.
Take $([x_1, \ldots, x_{n+1}], c) \in STR(G_0)$, $n \geq 2$. This implies that $c \in STR_2(G_0) = (\overline{L_0})_*$, therefore there are $u, v \in STR_2(G_0)$, uniquely determined, such that $c = [u, v]$. Thus $([x_1, \ldots, x_{n+1}], [u, v]) \in STR(G_0) = (\overline{K(G_0)})_\otimes$. It follows that there are the elements, uniquely determined, $d_1 = ([y_1, \ldots, y_s], \gamma_1) \in STR(G_0)$, $d_2 = ([z_1, \ldots, z_p], \gamma_2) \in STR(G_0)$ such that $(d_1, d_2) \in dom(\otimes)$ and

$$([x_1, \ldots, x_{n+1}], [u, v]) = d_1 \otimes d_2 \tag{11}$$

From $(d_1, d_2) \in dom(\otimes)$ we deduce that $y_s = z_1$ and

$$d_1 \otimes d_2 = ([y_1, \ldots, y_s, z_2, \ldots, z_p], [\gamma_1, \gamma_2]) \tag{12}$$

From (11) and (12) we deduce that

$$[x_1, \ldots, x_{n+1}] = [y_1, \ldots, y_s, z_2, \ldots, z_p] \tag{13}$$

$$[u, v] = [\gamma_1, \gamma_2]$$

We have $u, v, \gamma_1, \gamma_2 \in STR_2(G_0))$, $STR_2(G_0)$ is a $*$-Peano algebra and from $[u, v] = [\gamma_1, \gamma_2]$ we deduce $u = \gamma_1$ and $v = \gamma_2$. From (13) we deduce that $n+1 = s+p-1$ and $x_1 = y_1, \ldots, x_s = y_s$, $x_{s+1} = z_2, \ldots, x_{n+1} = z_p$. It follows that $d_1 = ([x_1, \ldots, x_s], u)$ and $d_2 = ([x_s, \ldots, x_{n+1}], v)$. But $d_1 \in STR(G_0)$ and $d_2 \in STR(G_0)$. We remark that s is uniquely determined. Thus the proposition is proved. ■

Proposition 6 *(splitting property II)*
If $([x_1, \ldots, x_{n+1}], c) \in ASP(\mathcal{G})$ and $n \geq 2$ then there are $u, v \in STR_2(G_0)$ and $k \in \{2, \ldots, n\}$, uniquely determined, such that
$c = [u, v]$
$([x_1, \ldots, x_k], u) \in ASP(\mathcal{G})$
$([x_k, \ldots, x_{n+1}], v) \in ASP(\mathcal{G})$

Proof. Consider $([x_1, \ldots, x_{n+1}], c) \in ASP(\mathcal{G})$ and $n \geq 2$. Because $ASP(\mathcal{G}) \subseteq STR(G_0)$ we can apply Proposition 5. Thus, there are $u, v \in STR_2(G_0)$ and $k \in \{2, \ldots, n\}$, uniquely determined, such that
$c = [u, v]$
$([x_1, \ldots, x_k], u) \in STR(G_0)$
$([x_k, \ldots, x_{n+1}], v) \in STR(G_0)$
But $h(c) \in L$, therefore from the definition of the mapping h we deduce that $\sigma(h(u), h(v)) \in L$. We have $h(u) \in (\overline{L_0})_\sigma$ and $h(v) \in (\overline{L_0})_\sigma$. From $L \in Initial((\overline{L_0})_\sigma)$ we deduce that $h(u) \in L$ and $h(v) \in L$. This shows that $([x_1, \ldots, x_k], u) \in ASP(\mathcal{G})$ and $([x_k, \ldots, x_{n+1}], v) \in STR(G_0)$. ■

5 Inference process based on accepted structured paths

We consider a stratified graph $\mathcal{G} = (G_0, L, T, u, f)$ over $G_0 = (S, L_0, T_0, f_0)$. Let $\mathcal{Y} = (Y, \odot)$ be a binary algebra and an injective mapping $ob : S \longrightarrow Y$. We suppose that for each $u \in L$ we have an algorithm $Alg_u : Y \times Y \longrightarrow Y$. This means that is a partial mapping such that $dom(Alg_u) \subseteq Y \times Y$ and for every pair $(x, y) \in dom(Alg_u)$ given as input for Alg_u) this algorithm gives as output some element of Y.

Definition 3 *A **knowledge processing system** based on stratified graphs is a tuple*

$$KPS = (\mathcal{G}, (Y, \odot), ob, \{Alg_u\}_{u \in L})$$

where

- *$G = (G_0, L, T, u, f)$ is a stratified graph over $G_0 = (S, L_0, T_0, f_0)$;*
- *$(Y, \odot)$ is a binary partial algebra;*
- *$ob : S \longrightarrow Y$ is an injective mapping;*
- *For each $u \in L$ the entity Alg_u is an algorithm that defines a mapping Alg_u : $dom(ALg_u) \longrightarrow Y$, where $dom(Alg_u) \subseteq Y \times Y$.*

We agree to say that such a structure is a **knowledge processing system over $\mathcal{G}$ with Y as output space** an we denote this property by $KPS(\mathcal{G}, Y)$.

For each $d = ([x_1, \dots, x_{n+1}], \sigma(u, v)) \in ASP(\mathcal{G})$ we consider the image $d_{ob} = ([ob(x_1), \dots, ob(x_{n+1})], \sigma(u, v))$ of the path d. We denote

$$ASP_{ob}(\mathcal{G}) = \{d_{ob} \mid d \in ASP(\mathcal{G})\}$$

We remark that we can consider the operation $\otimes$ for the case of images of accepted paths, as in the case of structured paths:

$$\otimes : ASP_{ob} \times ASP_{ob} \longrightarrow ASP_{ob}$$

as follows:

- $dom(\otimes) = \{((\alpha_1, u_1), (\alpha_2, u_2)) \mid (\alpha_1, u_1) \in ASP_{ob}, (\alpha_2, u_2) \in ASP_{ob},$
  $last(\alpha_1) = first(\alpha_2)\}$
- If $([x_1, \dots, x_k], u) \in ASP_{ob}$ and $([x_k, \dots, x_n], v) \in ASP_{ob}$ then

$$([x_1, \dots, x_k], u) \otimes ([x_k, \dots, x_n], v) = ([x_1, \dots, x_n], [u, v])$$

For a knowledge processing system based on stratified graphs we can define the inference process as in the next definition.

Definition 4 *The **inference process** $IP_{\mathcal{G},Y}$ generated by the stratified graph $\mathcal{G}$ and the output space Y is the mapping*

$$IP_{\mathcal{G},Y} : ASP_{ob}(\mathcal{G}) \longrightarrow Y$$

defined as follows:

$$IP_{\mathcal{G},Y}(d_{ob}) = \begin{cases} Alg_a(x, y) \text{ if } ([x, y], a) \in ASP_{ob} \\ \\ IP_{\mathcal{G},Y}(d_1) \odot IP_{\mathcal{G},Y}(d_2) \text{ if } d_{ob} = d_1 \otimes d_2 \end{cases}$$

Based on previous concepts and results we can propose the following algorithm of the inference process.
Input:
$KPS = (\mathcal{G}, (Y, \odot), ob, \{Alg_u\}_{u \in L})$; $((x, y) \in ob(S) \times ob(S)$
Method:
Compute $C = \{d_{ob} = (X, u) \mid first(X) = x, last(X) = y\}$;
Output:
$IP_{\mathcal{G},Y}(C)$
End

6 Conclusions

In this paper we treat from the mathematical point of view the concept of inference based on stratified graphs. We define the concept of knowledge processing system with stratified graphs and the concept of inference of such systems.

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